Admin

- Use slack for communication with me and each other
- In class **quiz** next **monday**
- Wednesday we discuss the quiz
- *Office hours will be once a week in an Asia friendly time: 6pm ?*
- I have been adding to the project doc. Start now if you like
- We will do mentor sign up this week
- **Any admin questions for me?**

About me …

What is Reinforcement Learning?

- Agent-oriented learning—learning by interacting with an environment to **achieve a goal**
- Learning by trial and error, with only **delayed evaluative feedback** (reward)
  - the kind of machine learning like natural learning (animals)
  - learning that can tell for itself when it is right or wrong

Plan for today

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Goal: Remind you about RL & sequential decision making

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- The basics of RL:
  - states, actions, rewards, time, MDPs, policies, value

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- The basics of RL:
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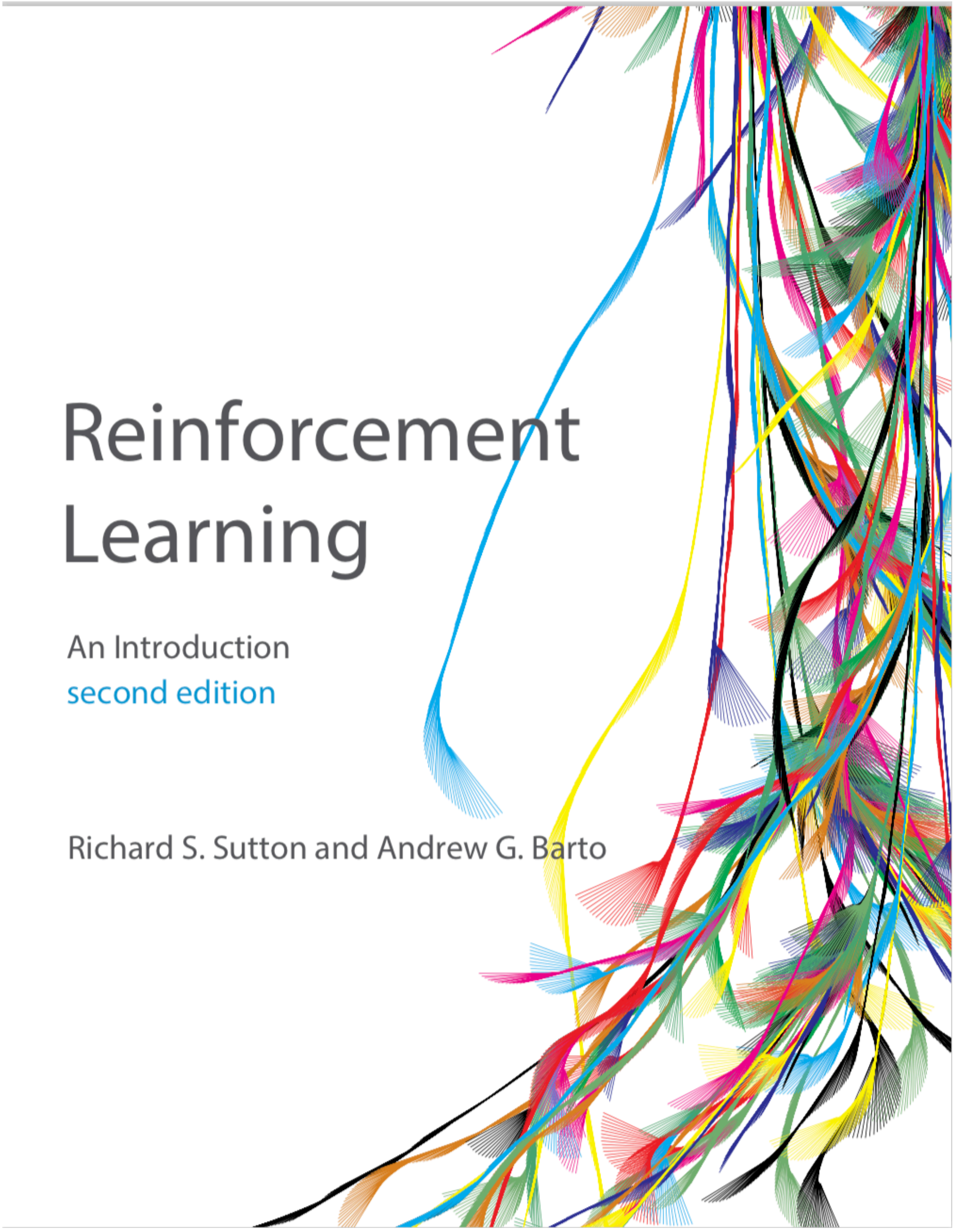
- Function approximation in RL
- Comments throughout on open research challenges, particularly related to learning in the real world!

Many ways to learn about RL



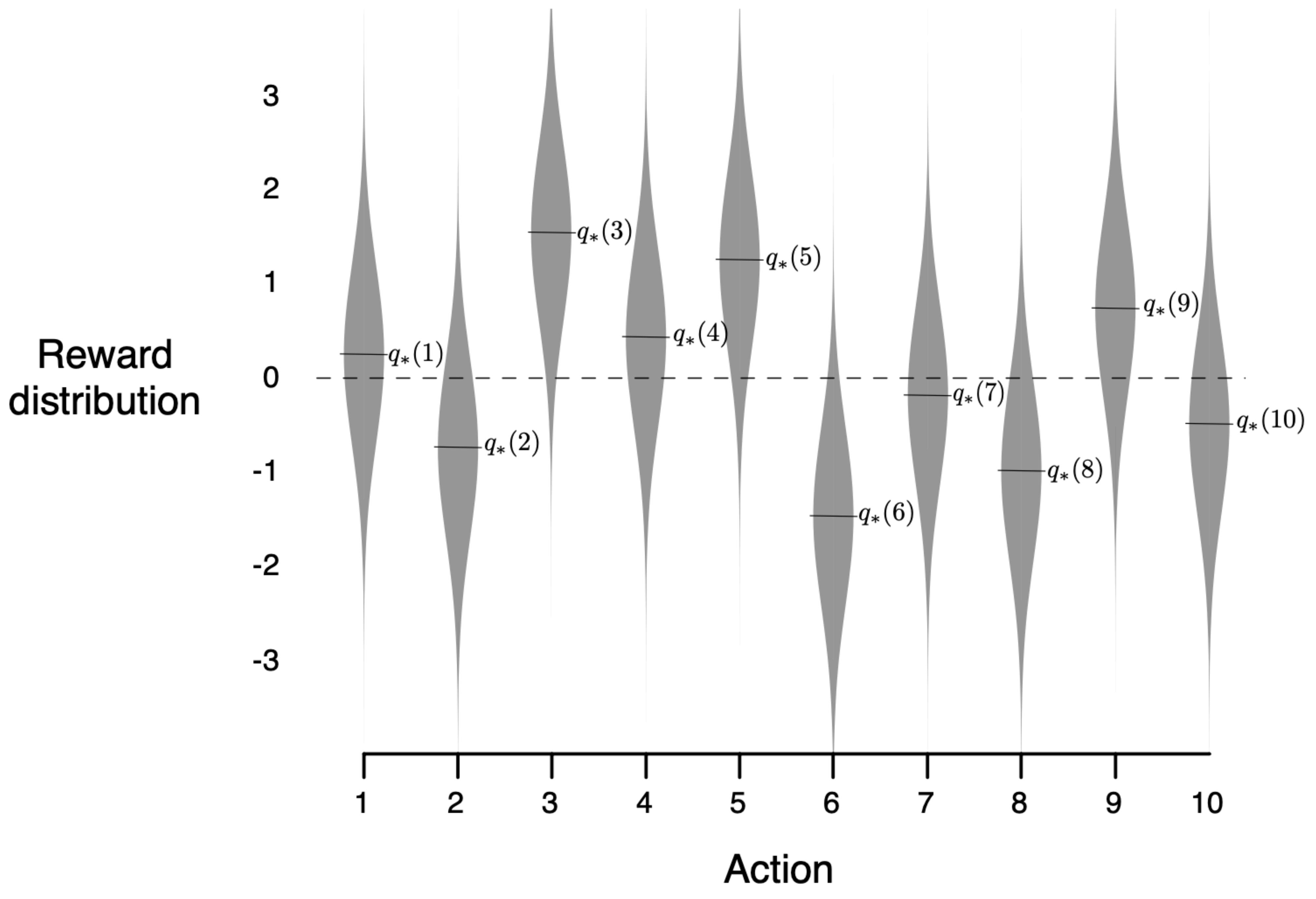
2nd edition: free and online

4 course RL specialization
(uab.ca/RLMOOC)

Key characteristics of RL

- **Evaluative feedback (reward)**
- Delayed consequences
- Must associate different actions with different situations
- Online and Incremental learning
- Need for trial and error, to explore as well as exploit
- Non-stationarity

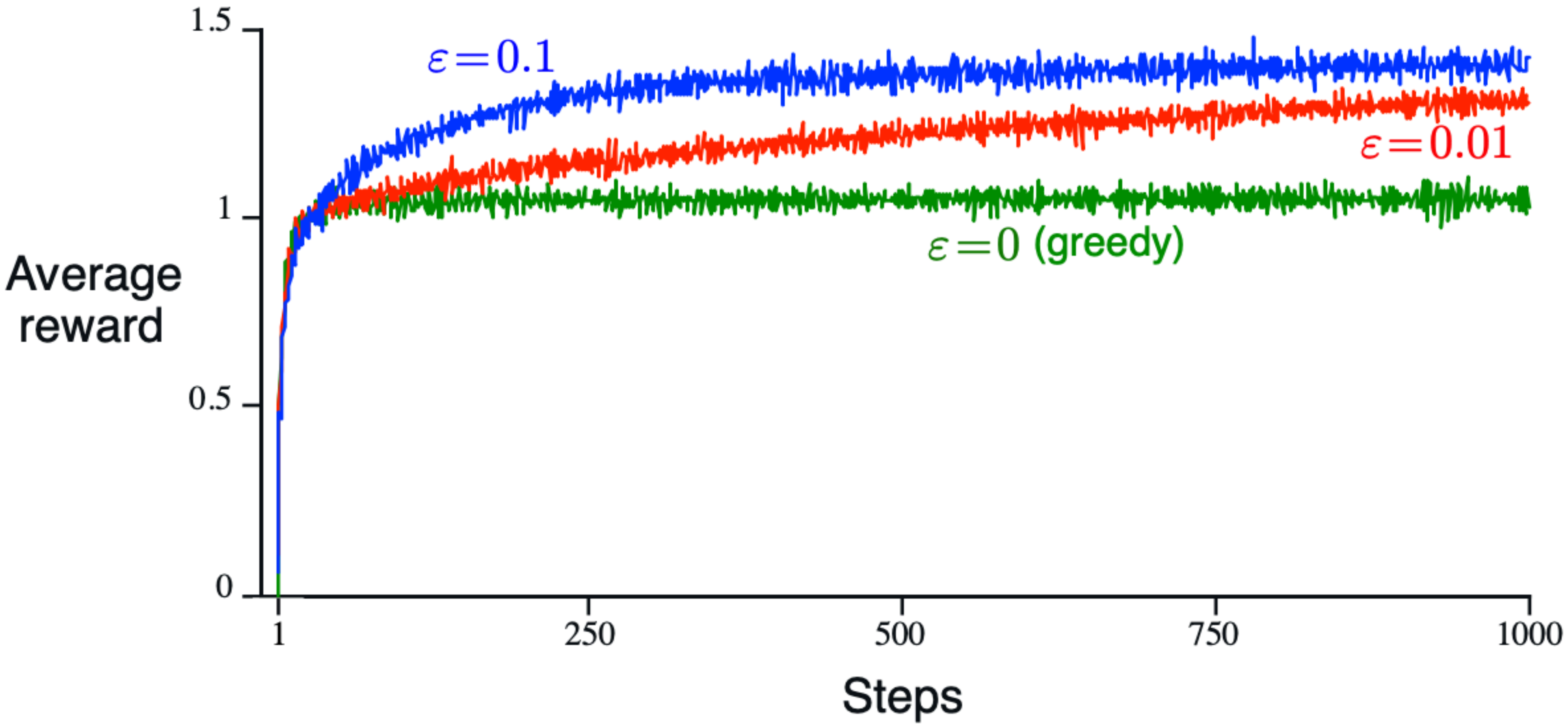
Multi-armed bandits

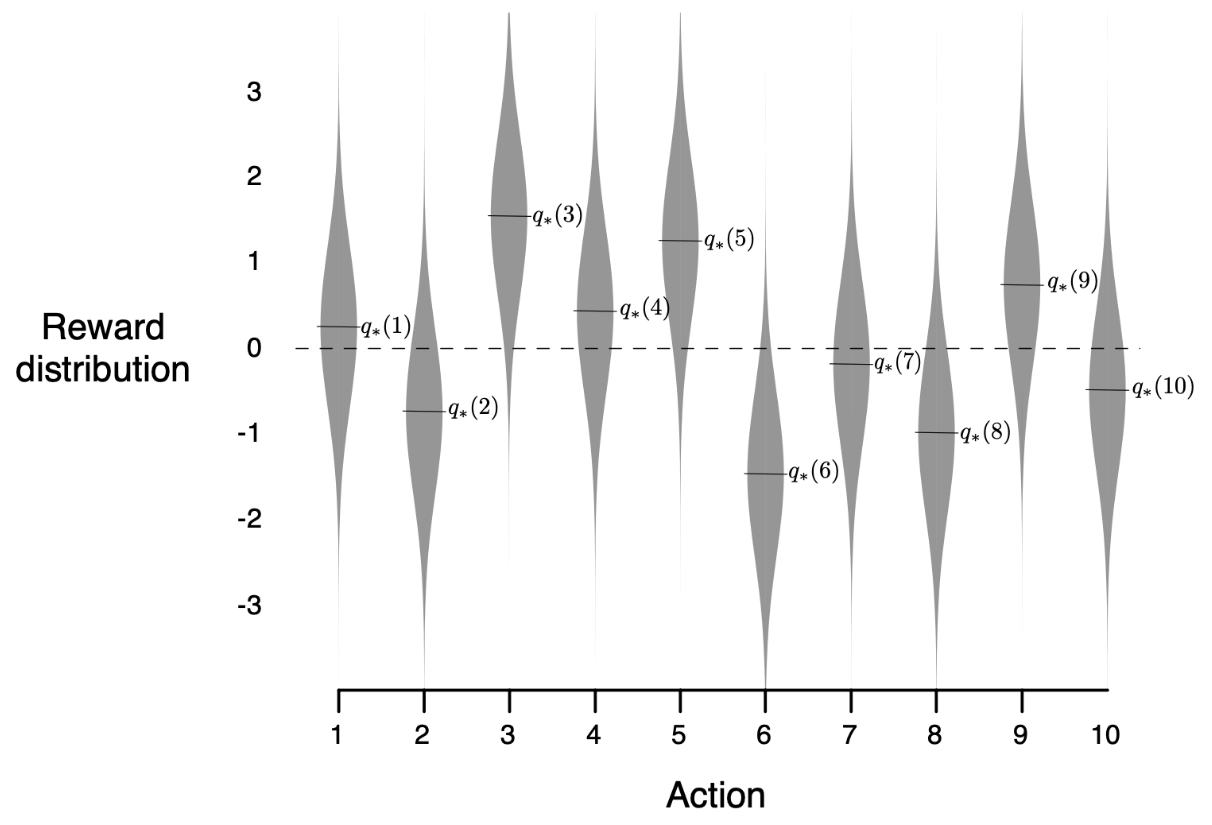


$$q_*(a) \doteq \mathbb{E}[R_t \mid A_t = a]$$

- Estimate the value of each action in order to find the best
- Only get samples of the reward by trying an action: **rewards of arms not chosen are not revealed**
- Means we need to try each arm enough, but we also don't want to suffer too much loss of potential reward
  - **=> the exploration / exploitation tradeoff**

Learning in a multi-armed bandit





$$q_*(a) \doteq \mathbb{E}[R_t \mid A_t = a]$$

$$Q_t(a) \doteq \frac{\text{sum of rewards when } a \text{ taken prior to } t}{\text{number of times } a \text{ taken prior to } t} = \frac{\sum_{i=1}^{t-1} R_i \cdot \mathbb{1}_{A_i=a}}{\sum_{i=1}^{t-1} \mathbb{1}_{A_i=a}}$$

$$A_t \doteq \underset{a}{\arg\max}\, Q_t(a)$$

Dimensions of learning revealed by the MAB problem

- The need to learn online and incrementally

$$NewEstimate \leftarrow OldEstimate + StepSize \Big[Target - OldEstimate \Big]$$

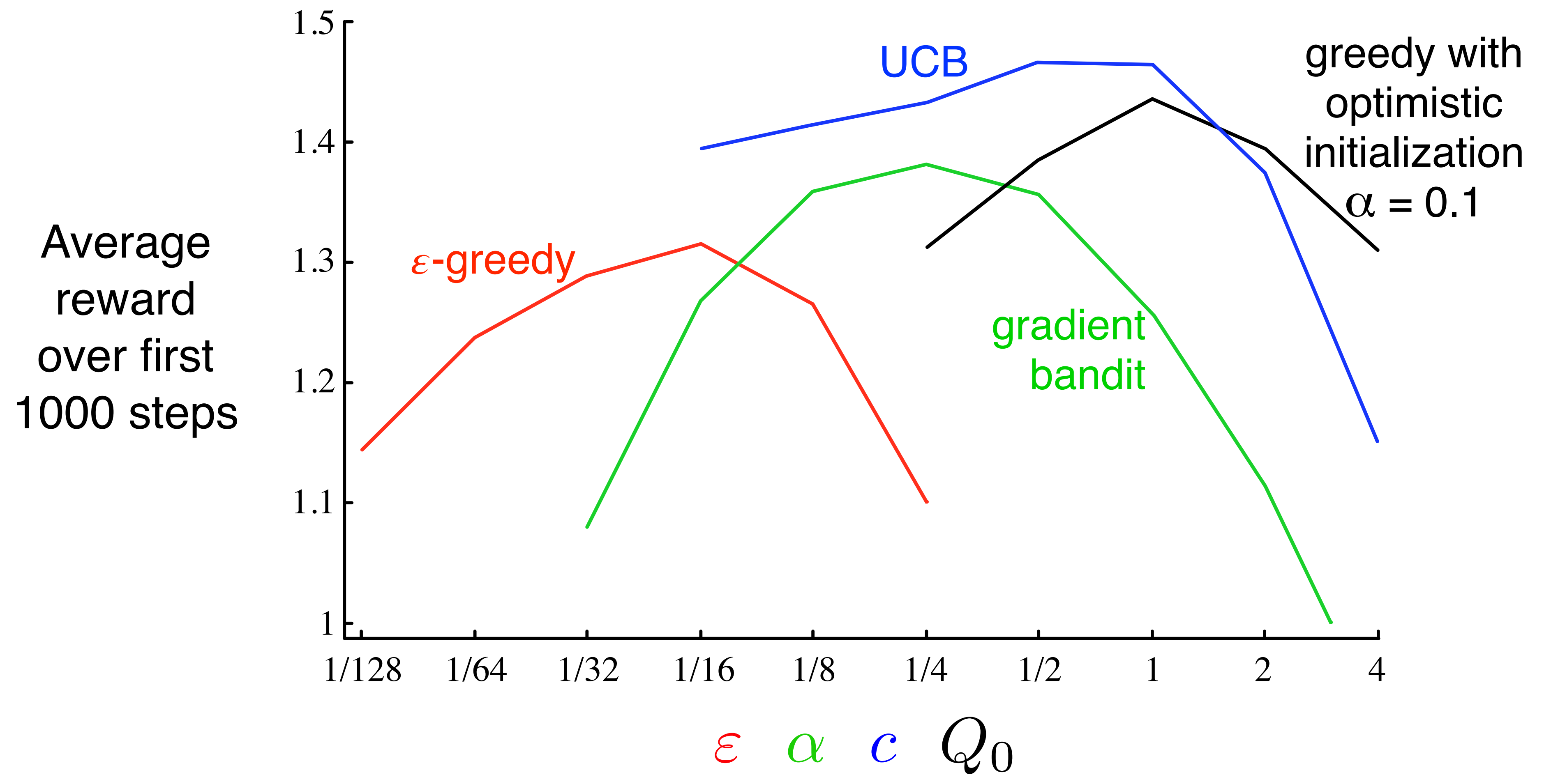
- Tracking and non-stationary tasks

$$Q_{n+1} \doteq Q_n + \alpha \Big[R_n - Q_n \Big]$$

- The role of initializing algorithms (e.g., optimistic init)

- Role of exploration algorithms (e.g., OI, e-greedy, UBC)

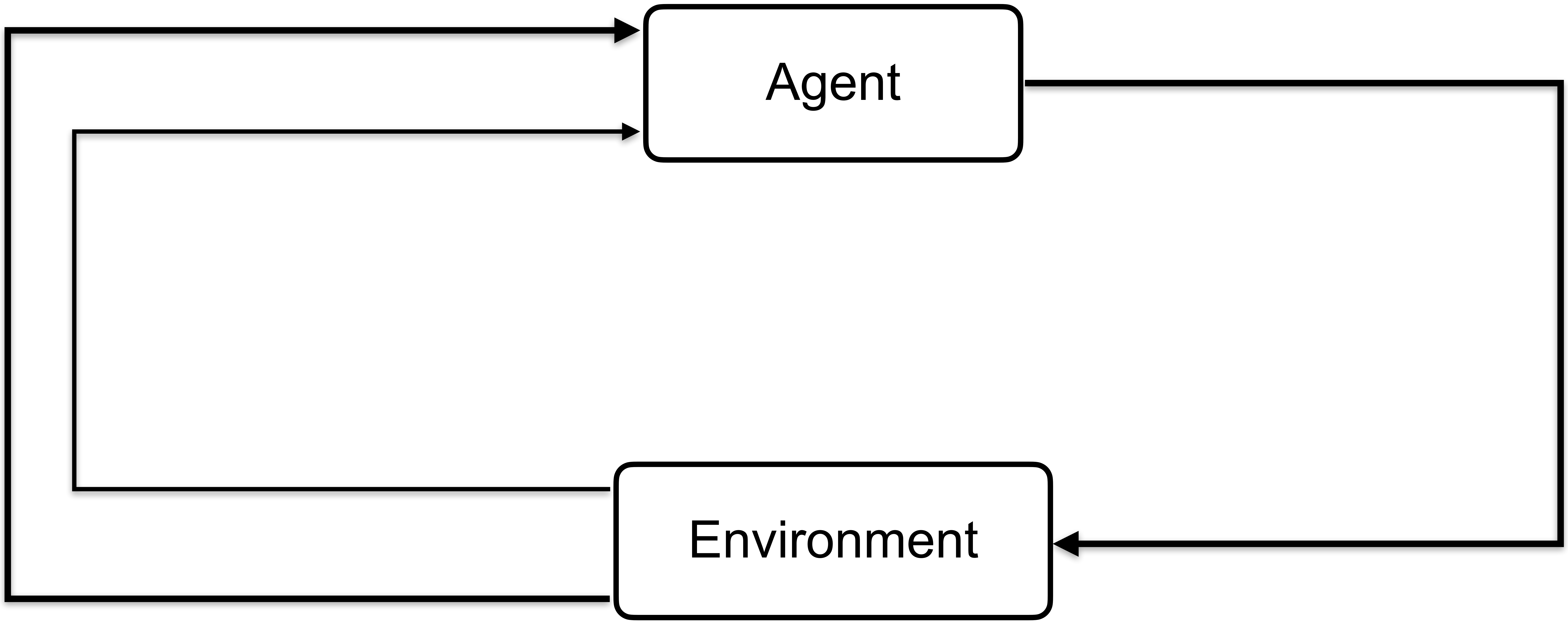
- Gradient methods



From Bandits to MDPs

- The k-armed bandit task shares some of the same key characteristics of the RL problem:
  - **Evaluative feedback (reward)**
  - Online and incremental learning
  - Need for trial and error, to **explore** as well as exploit
  - Non-stationary???
- Let's see how Markov Decision Processes and the RL problem differ from Bandits

The RL Interface



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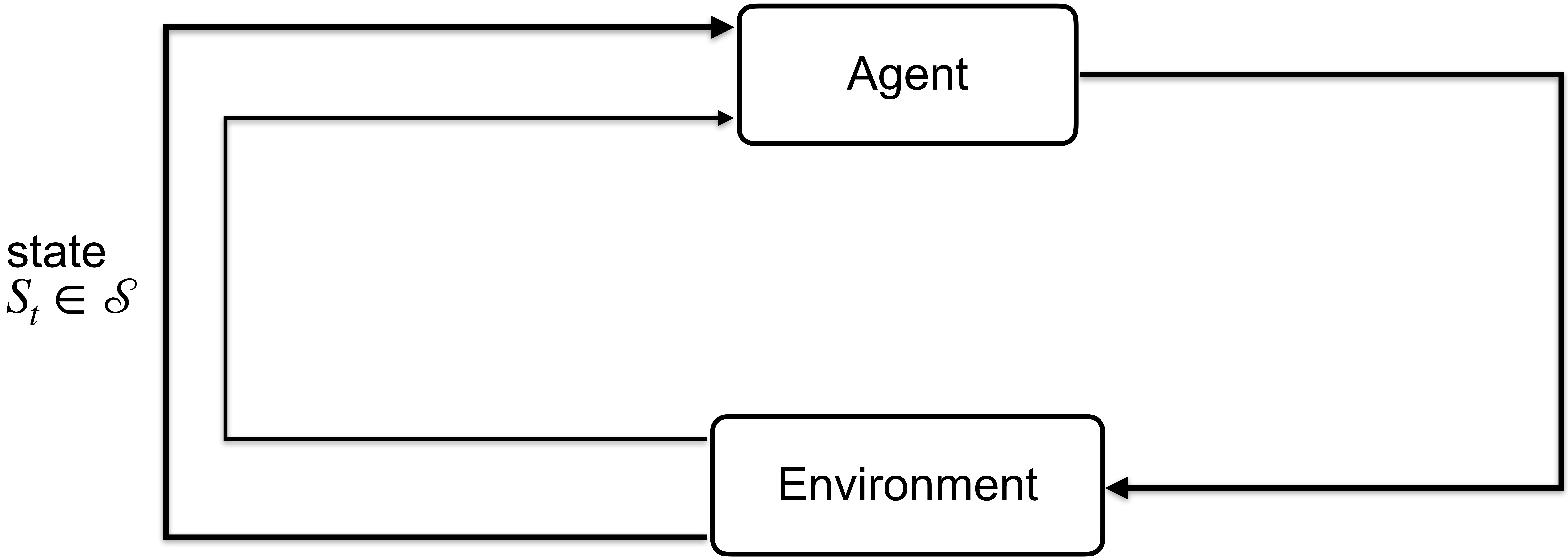

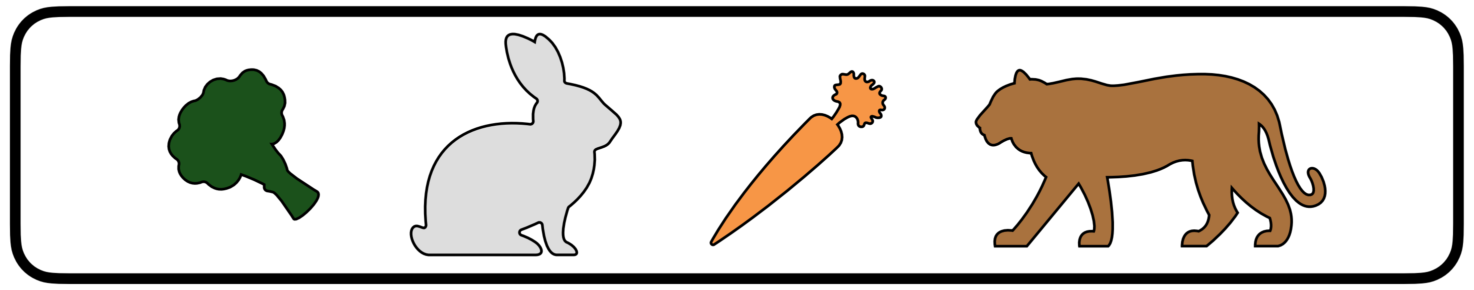

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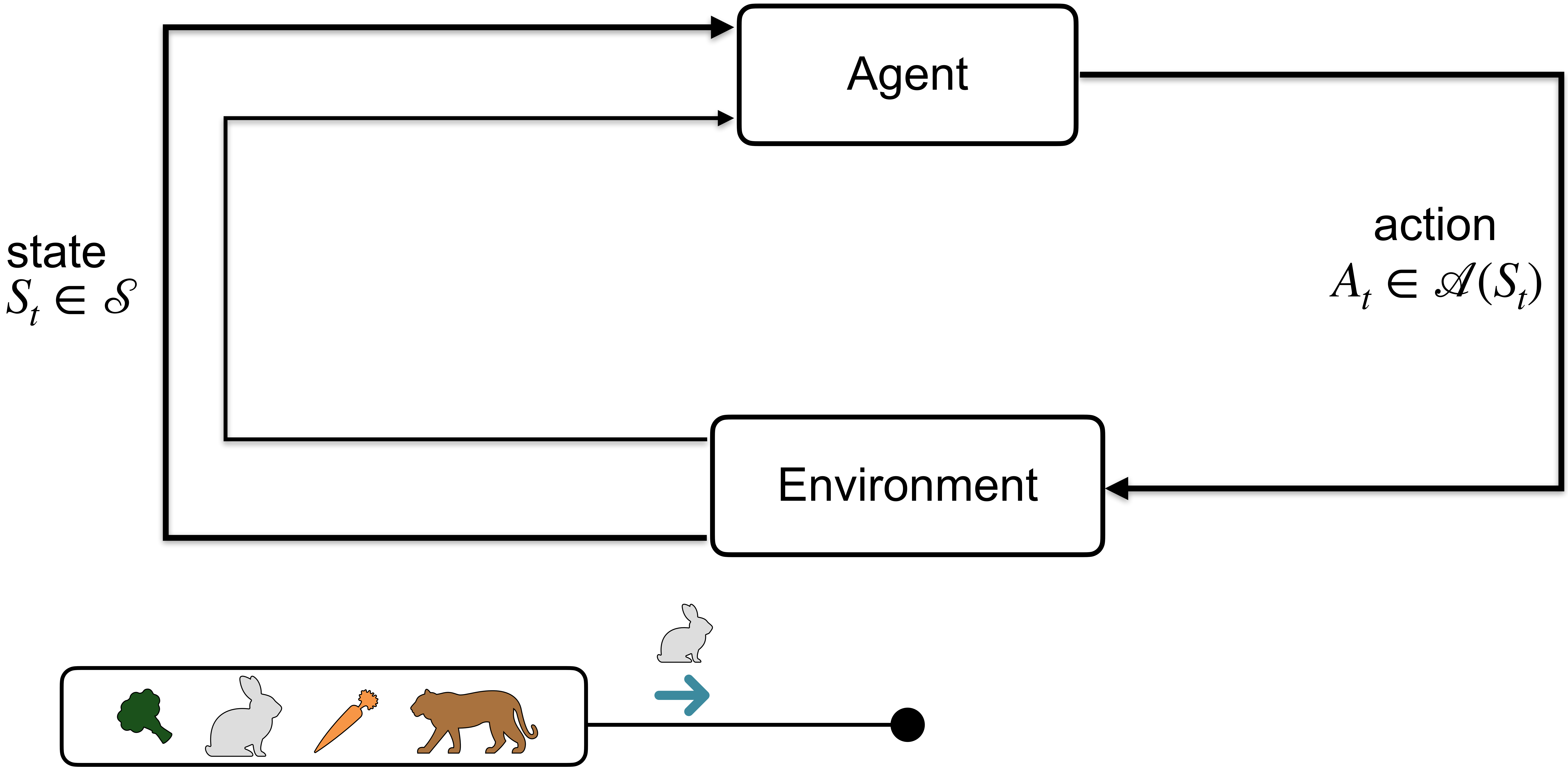

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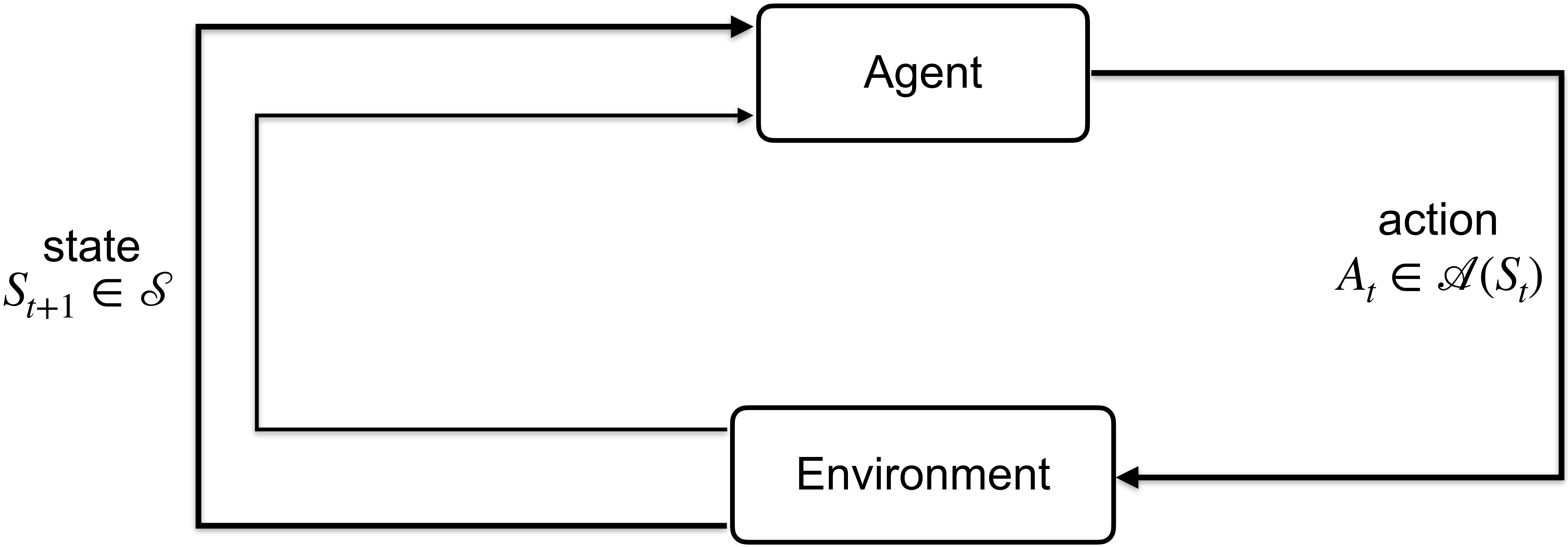

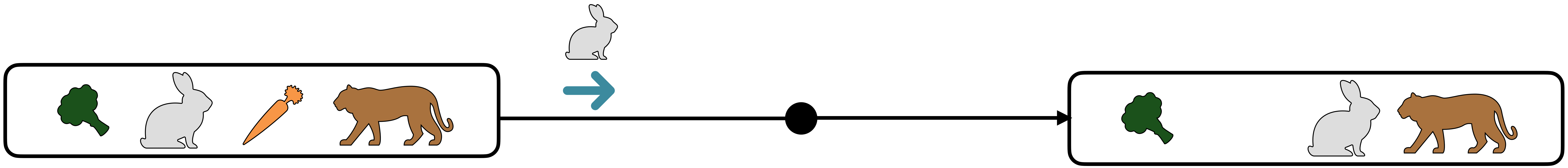

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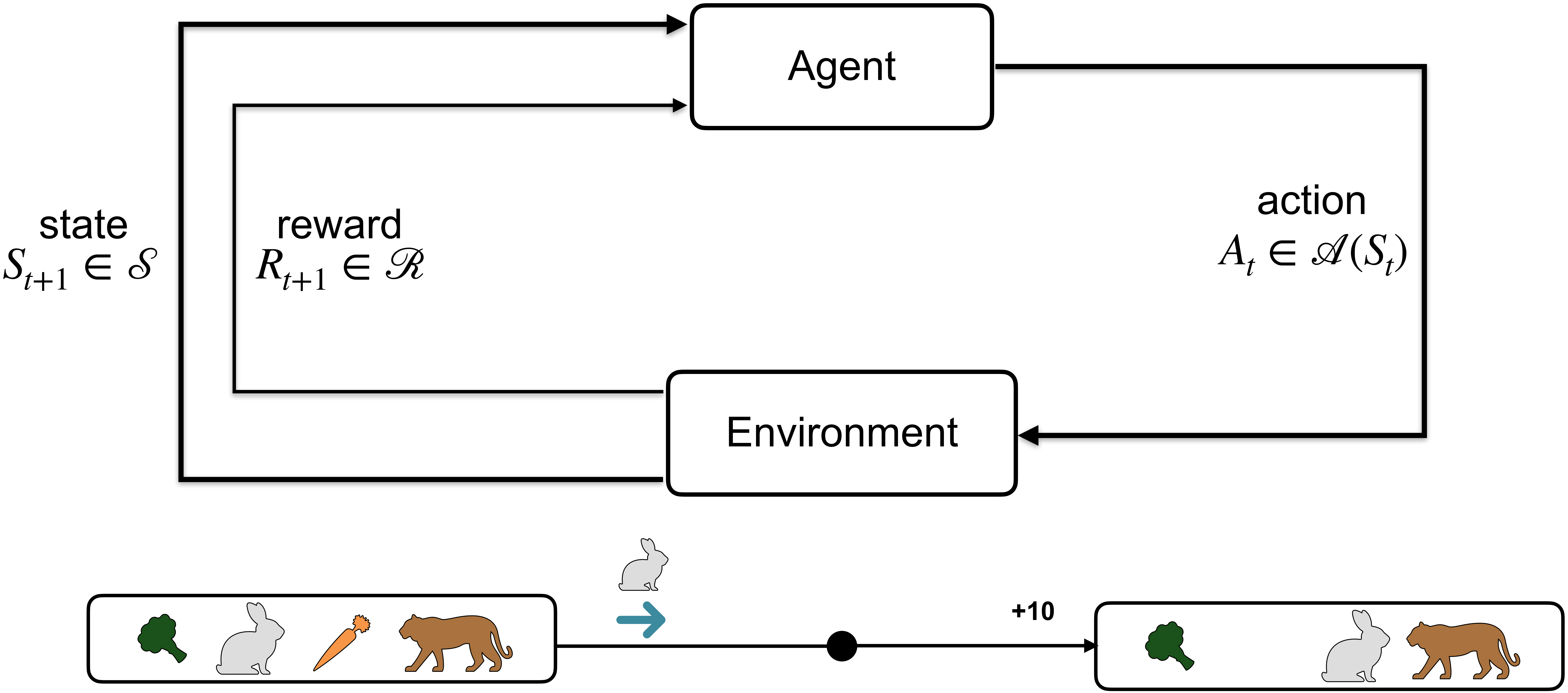

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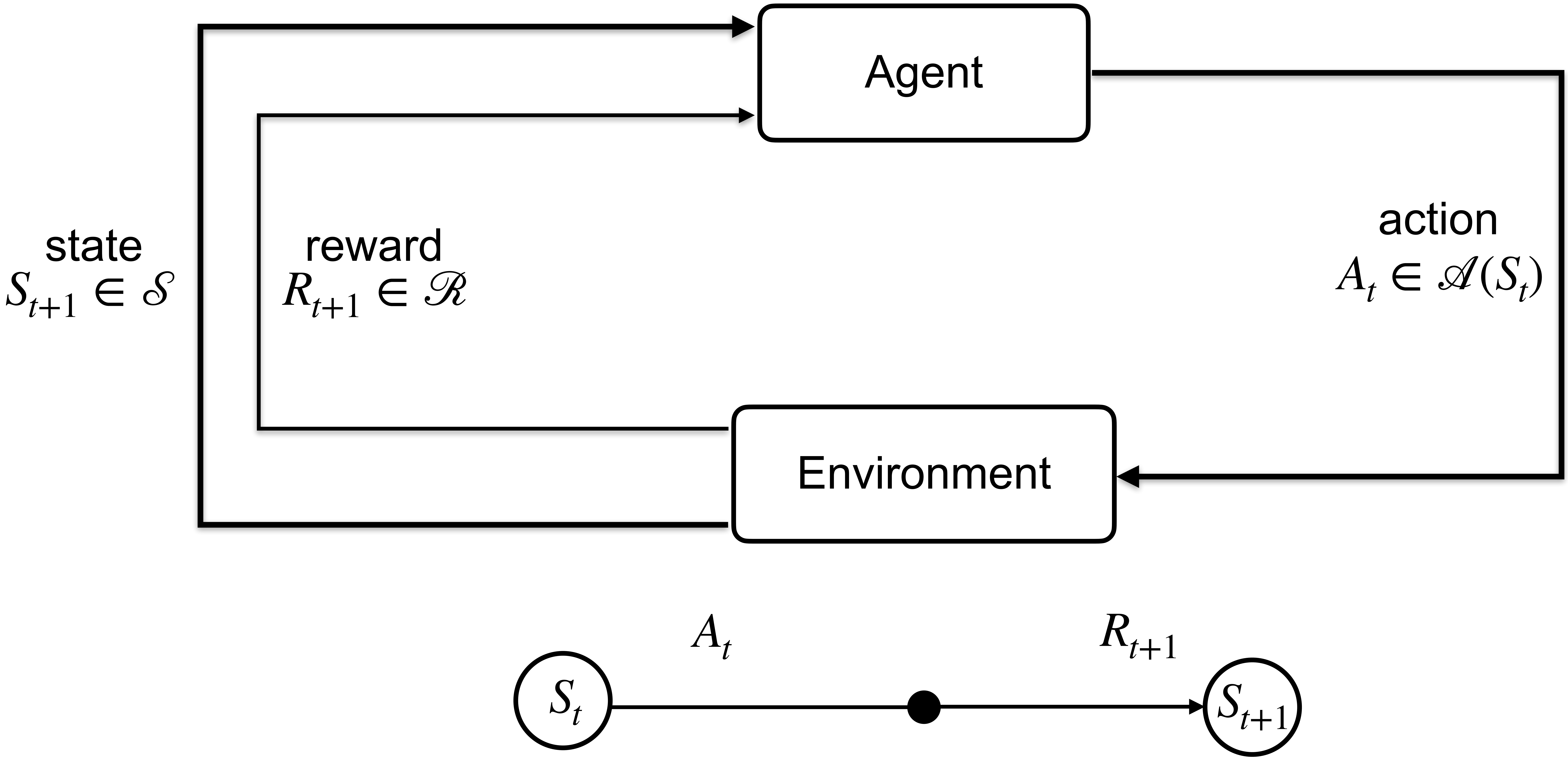
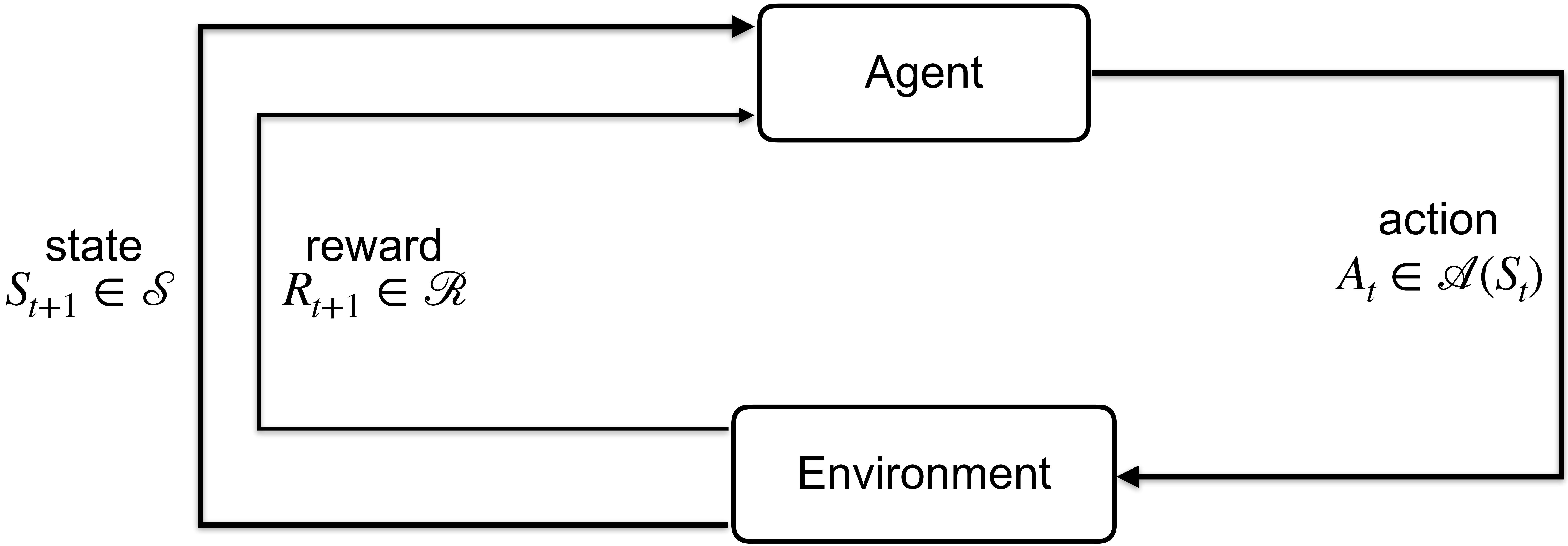


The interaction generates a stream of experience!



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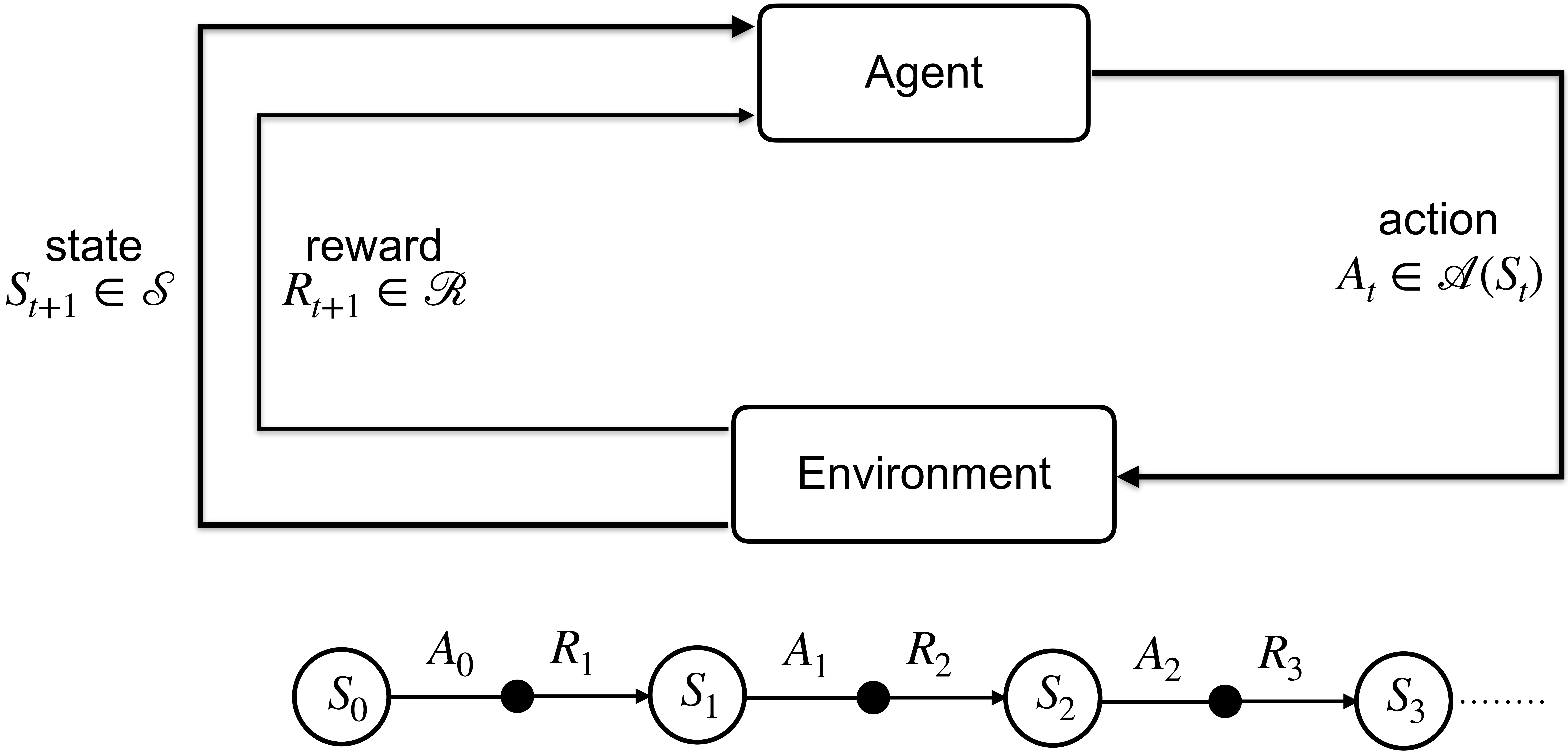

Finite Markov Decision Processes

- Environment may be unknown, stochastic and complex
  - we formalize this with the language of MDPs
- An RL problem is a finite MDP if:
  - the set of states, actions, and rewards are finite
  - there is a transition function that describes the probabilities of all possible next state S', and reward R
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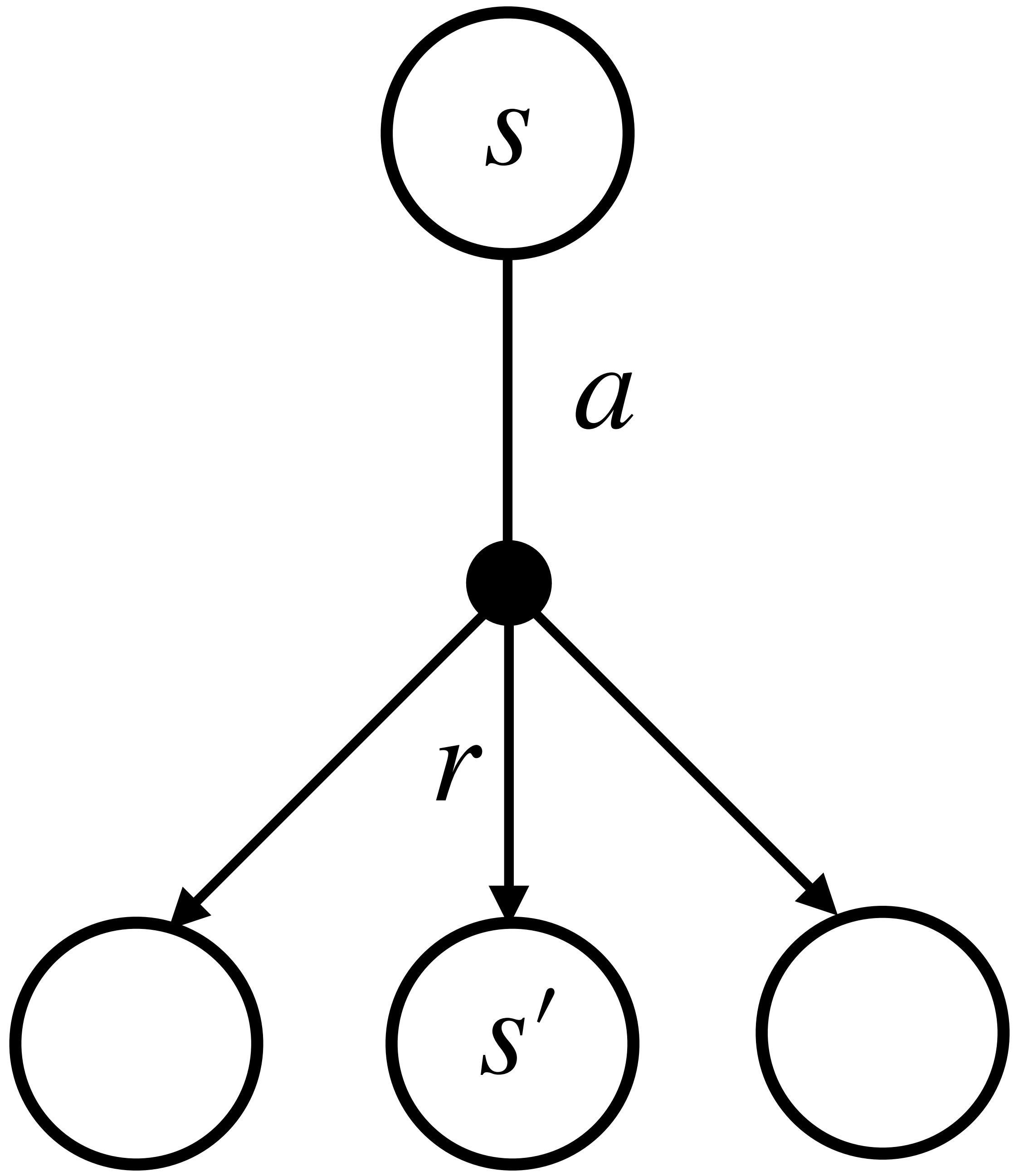
The dynamics of an MDP



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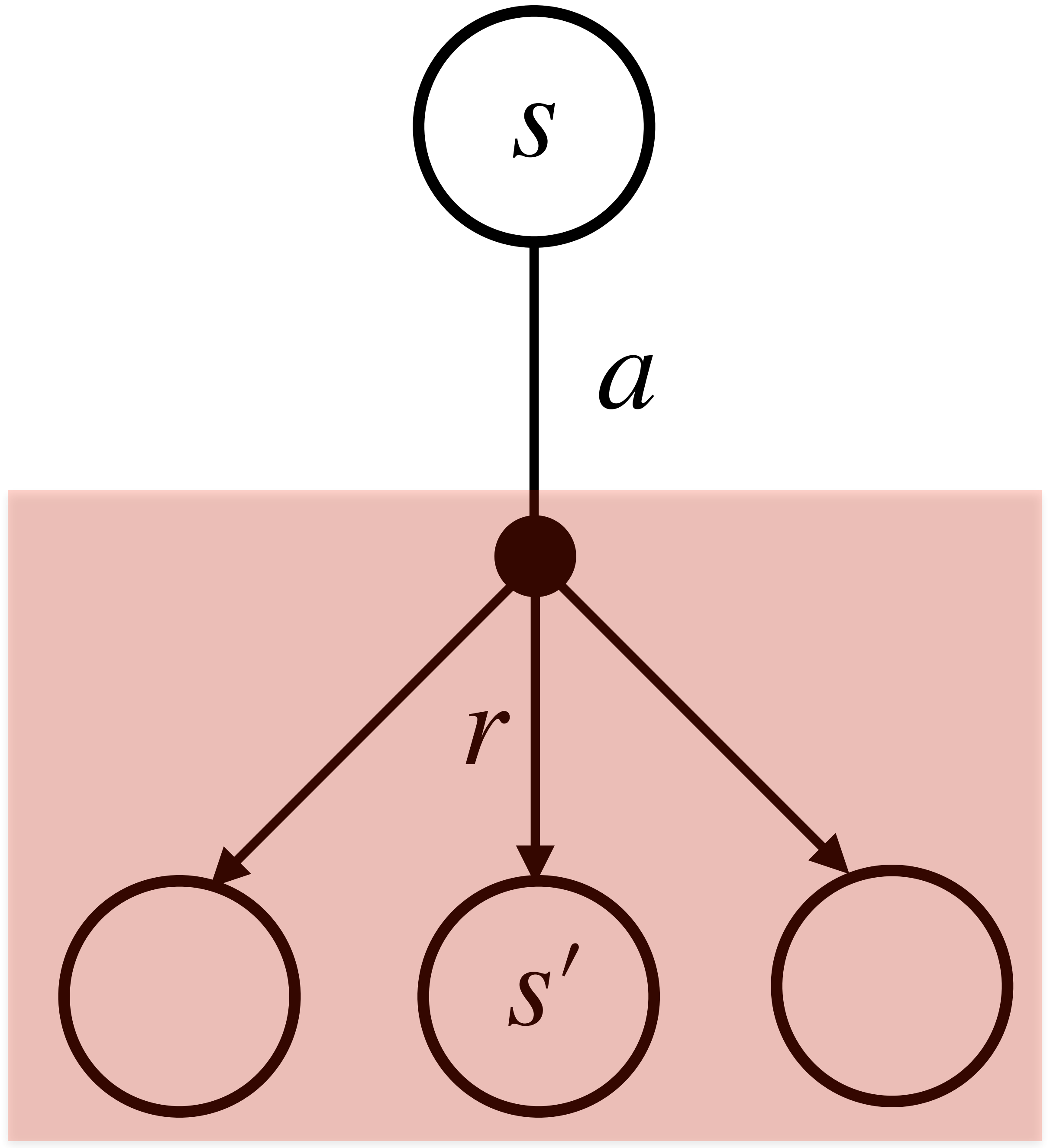

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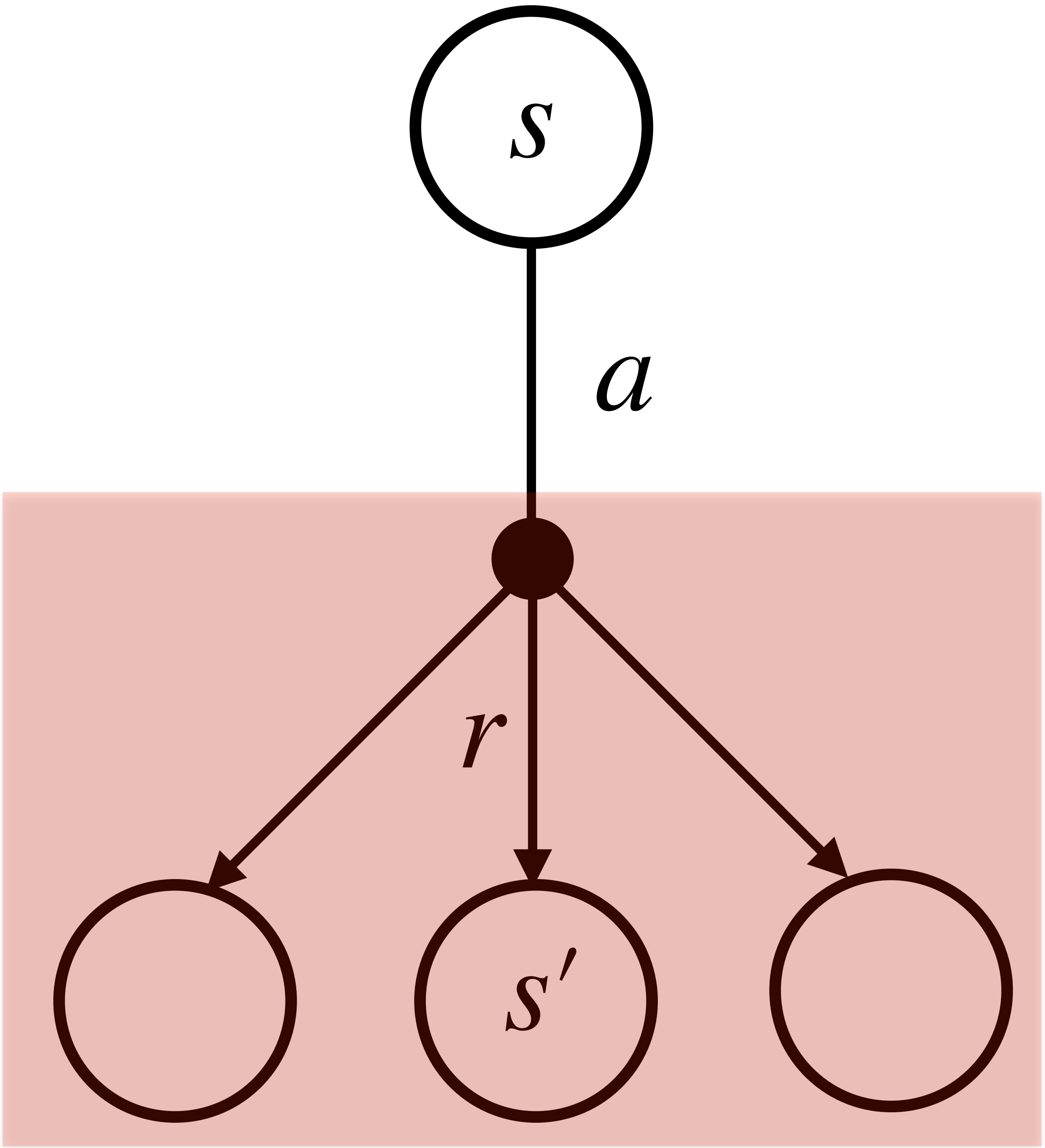

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$$p : \mathcal{S} \times \mathcal{R} \times \mathcal{S} \times \mathcal{A} \to [0, 1]$$

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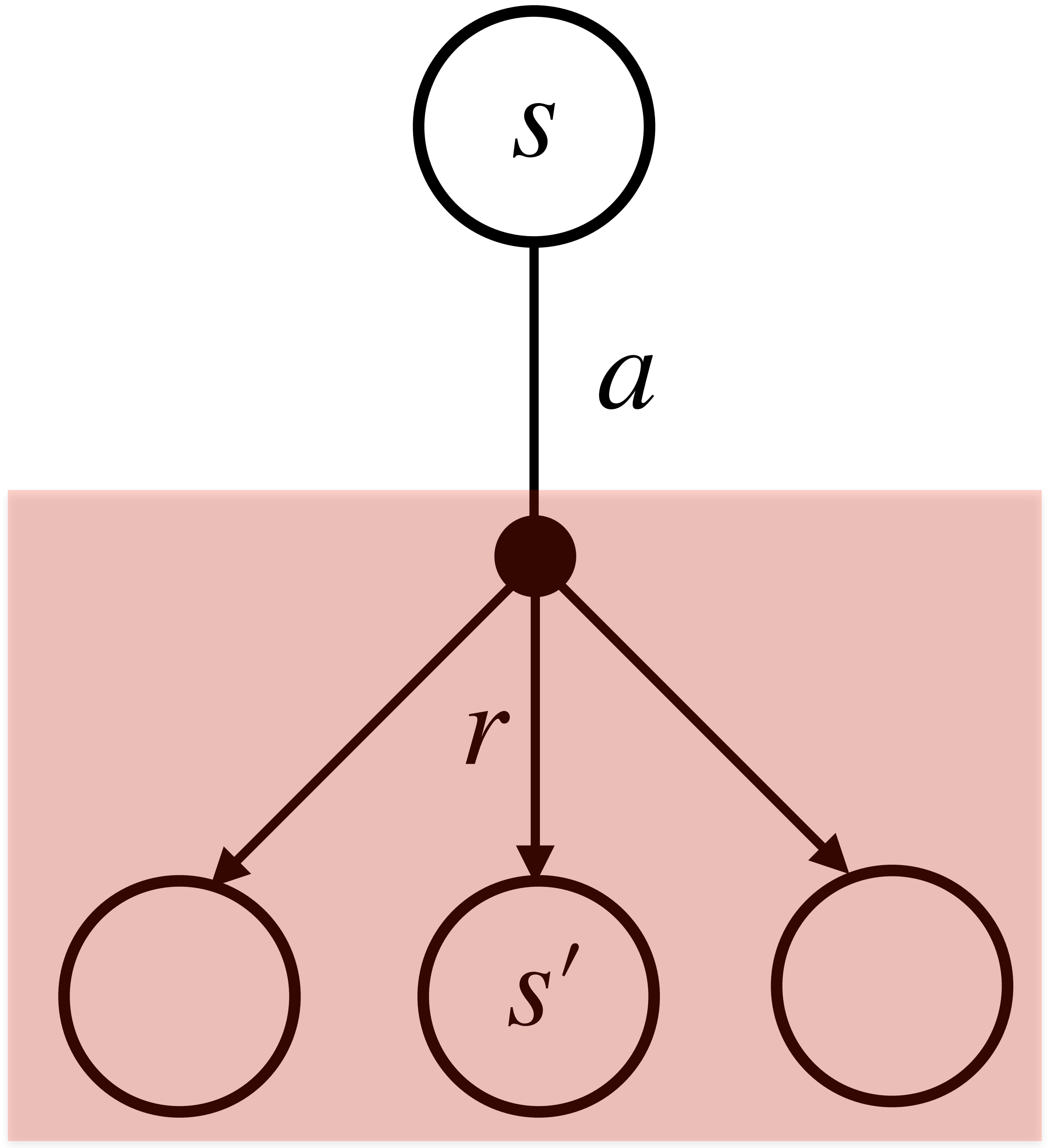

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Remembering earlier states would not improve predictions about the future

The goal of life: more reward

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- But, the agent's interaction may never end, so we discount rewards far into the future

$$G_t \doteq R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \ldots + \gamma^{k-1} R_{t+k} + \ldots$$

$$= \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$

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- In each state, the agent should choose the action that results in the highest return, **in expectation—why the expectation?**

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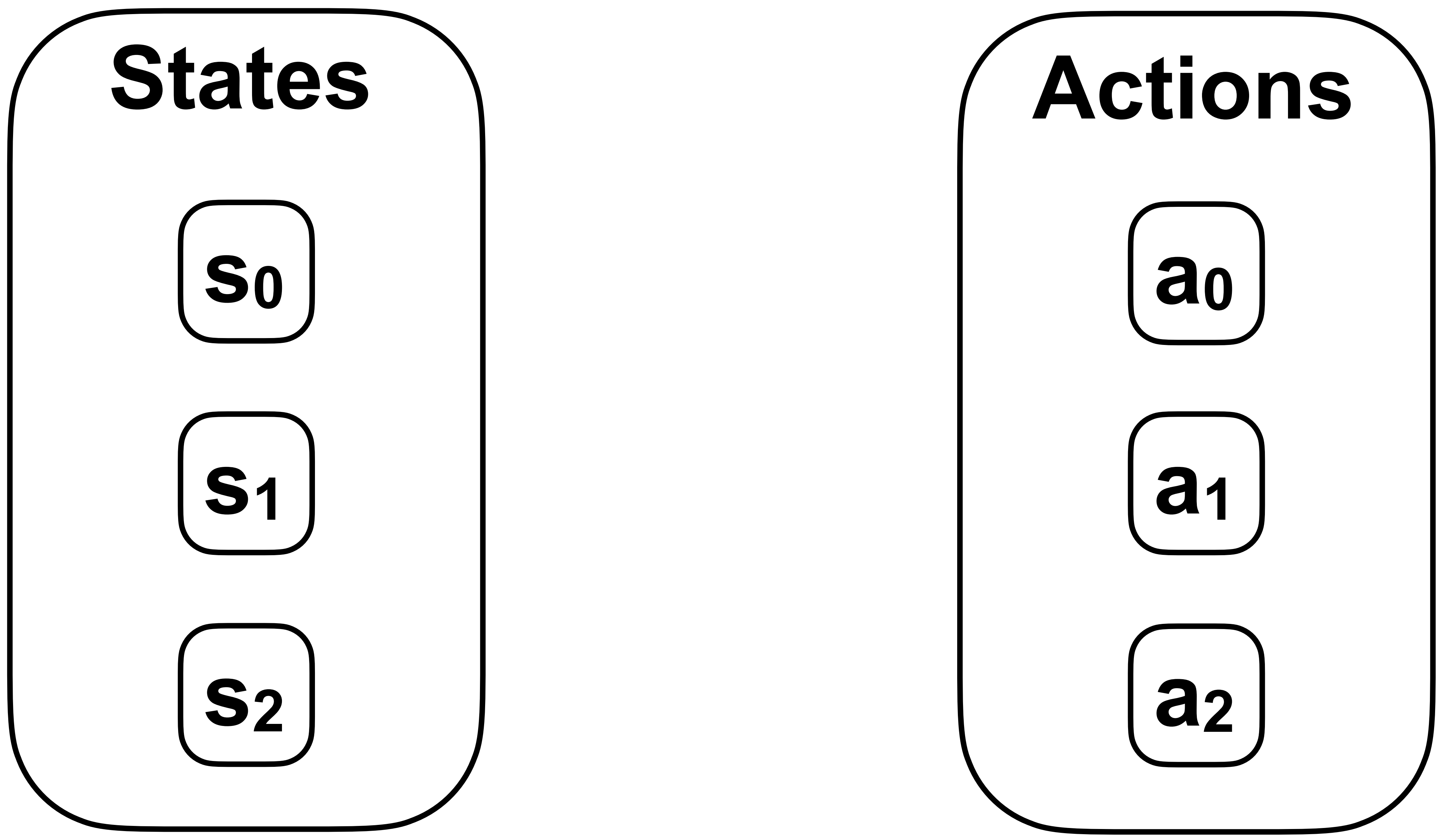
Policies

- Deterministic policy

$$\pi(s) = a$$



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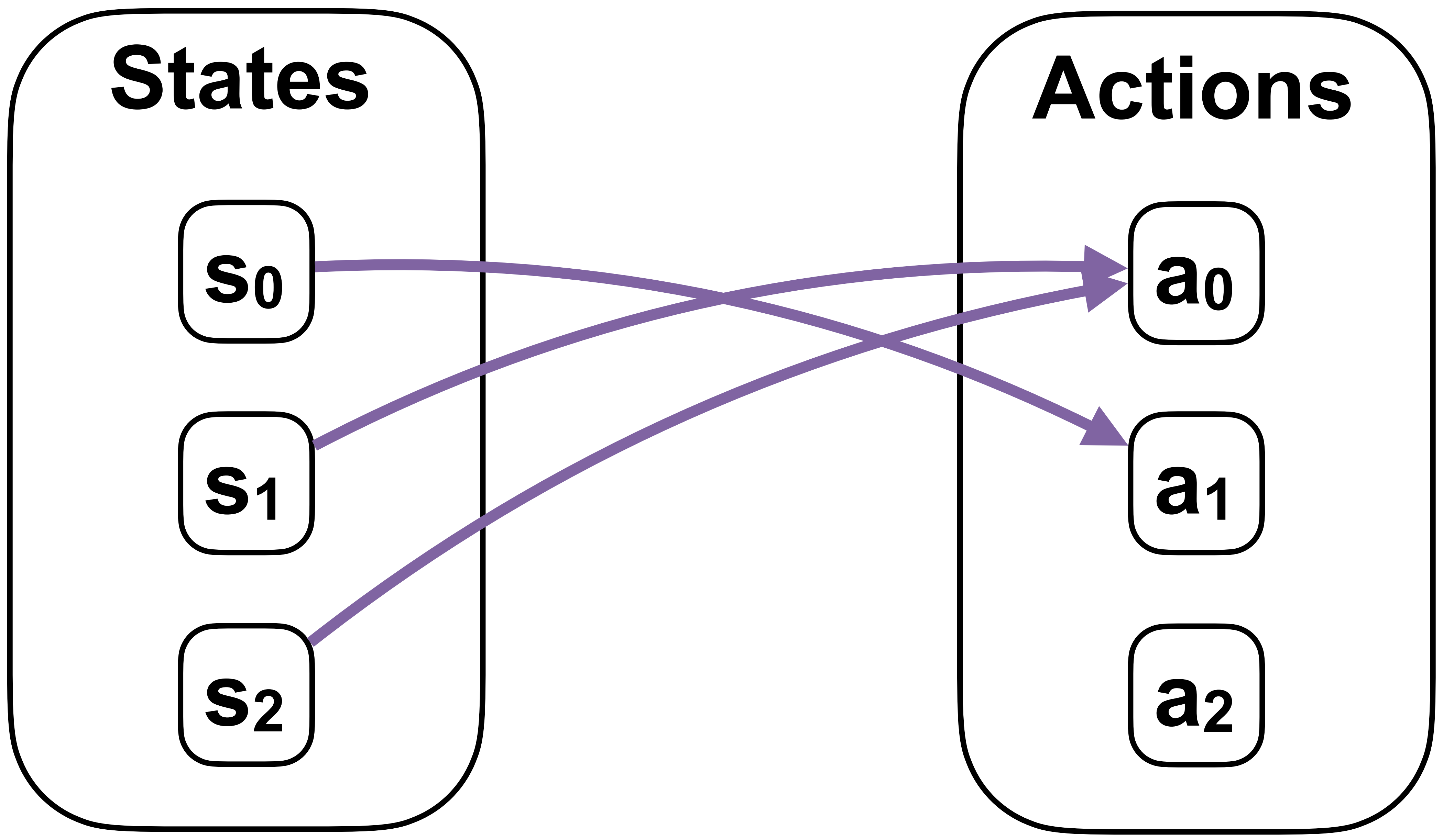

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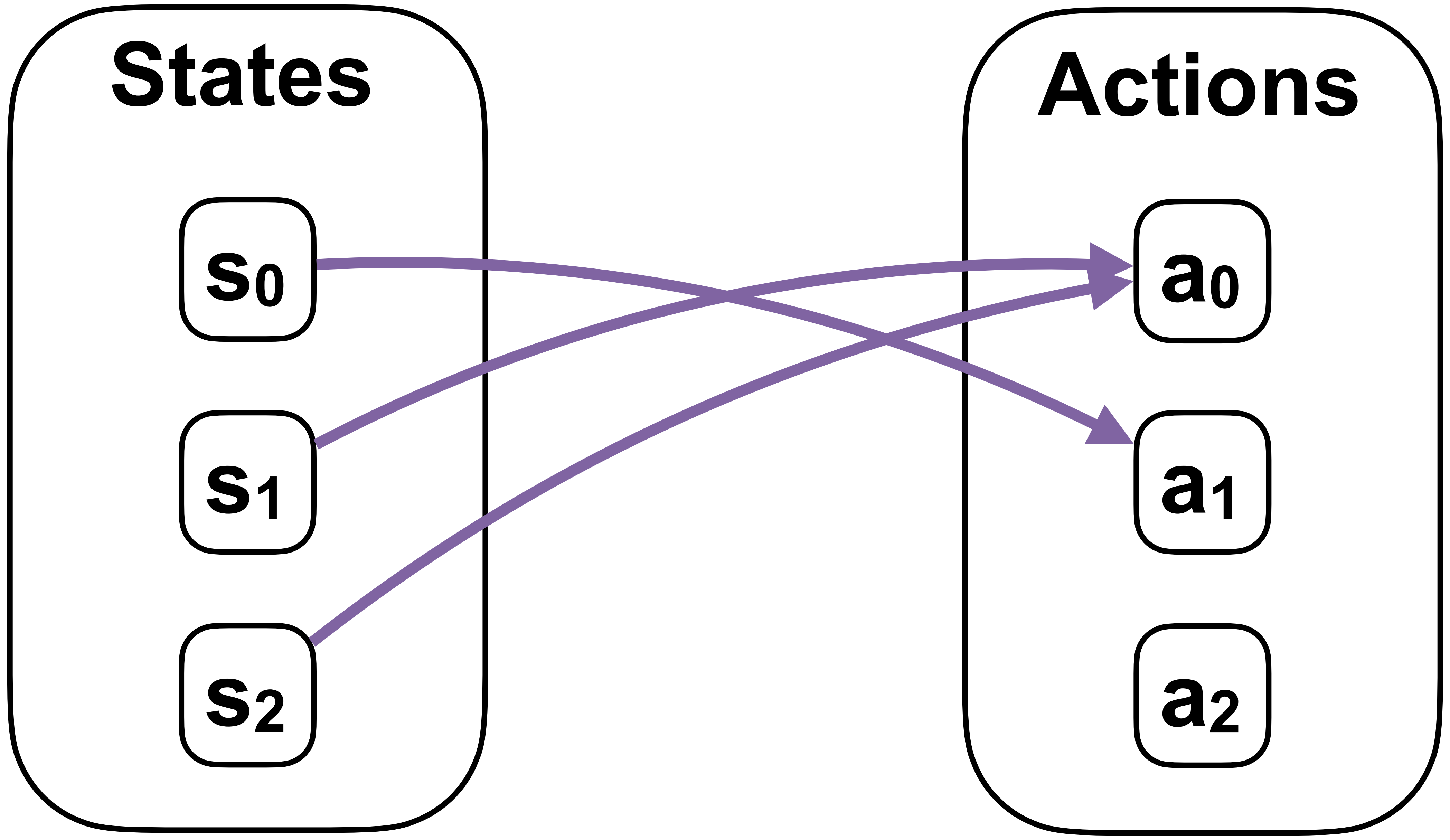

State	Action
s_0	a_1
s_1	a_0
s_2	a_0

Policies

- Stochastic policy:

$$\pi(a \mid s)$$

- where $\sum_{a \in \mathscr{A}(s)} \pi(a \mid s) = 1$

- and $\pi(a \mid s) \geq 0$

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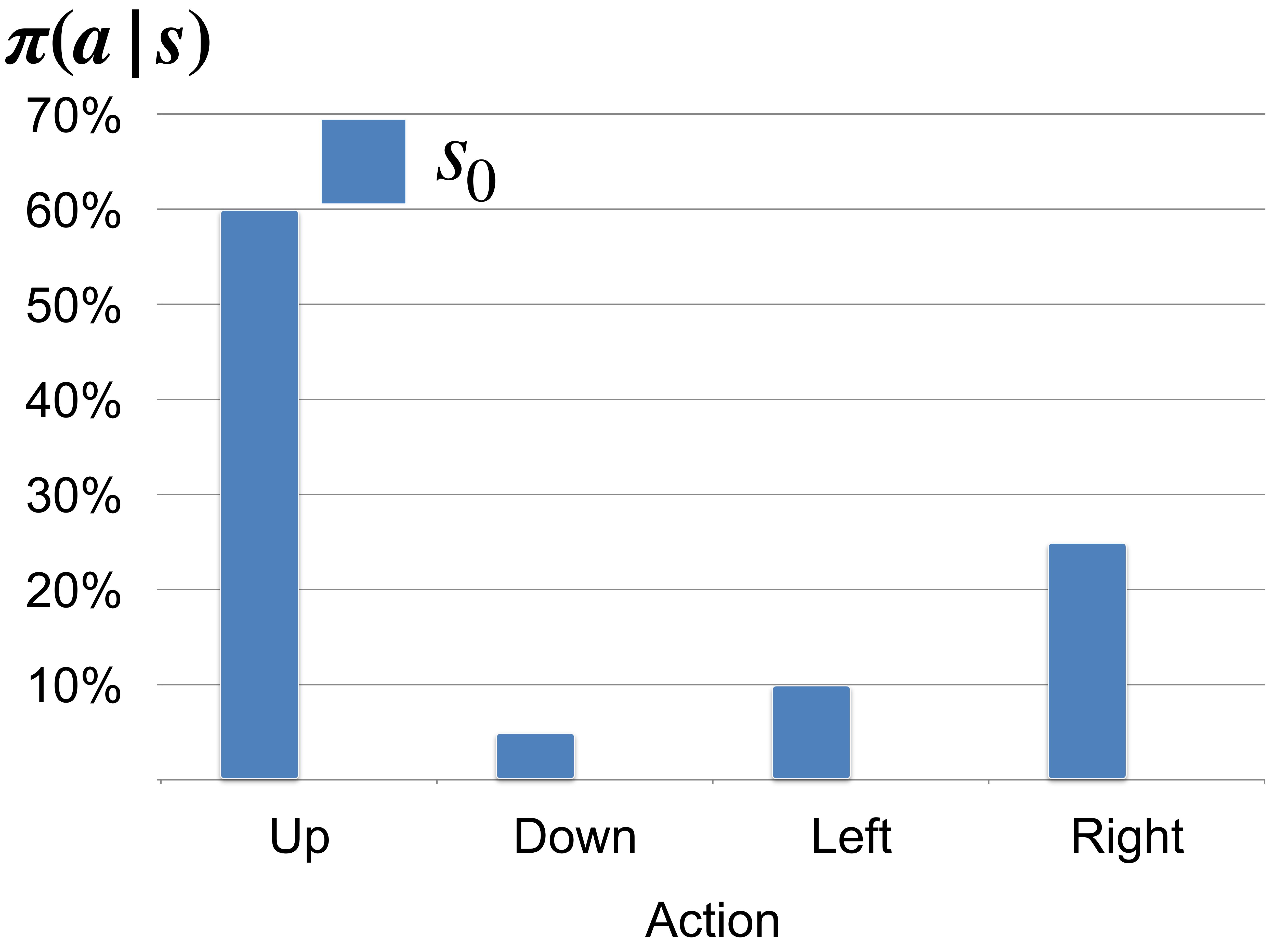

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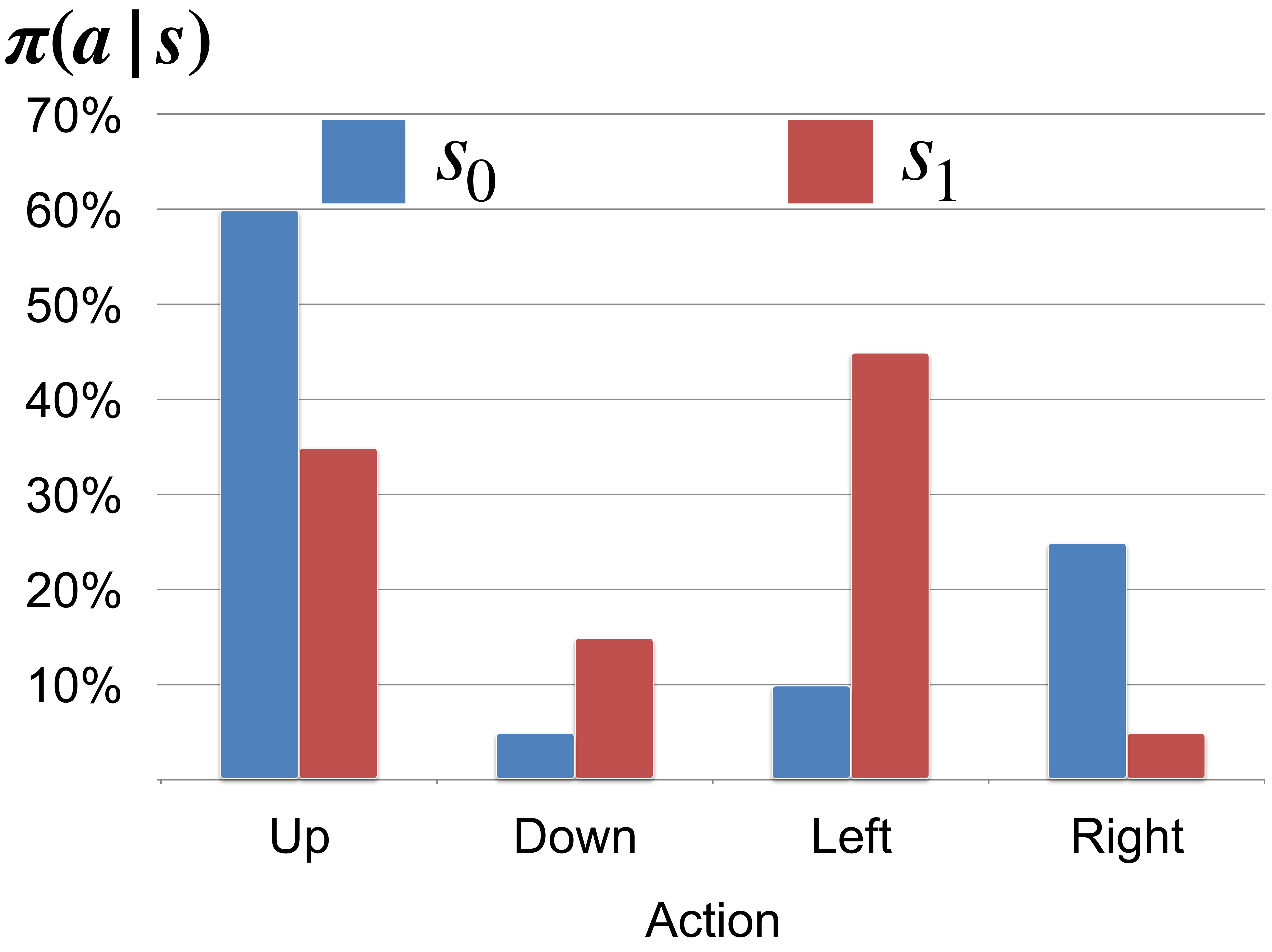
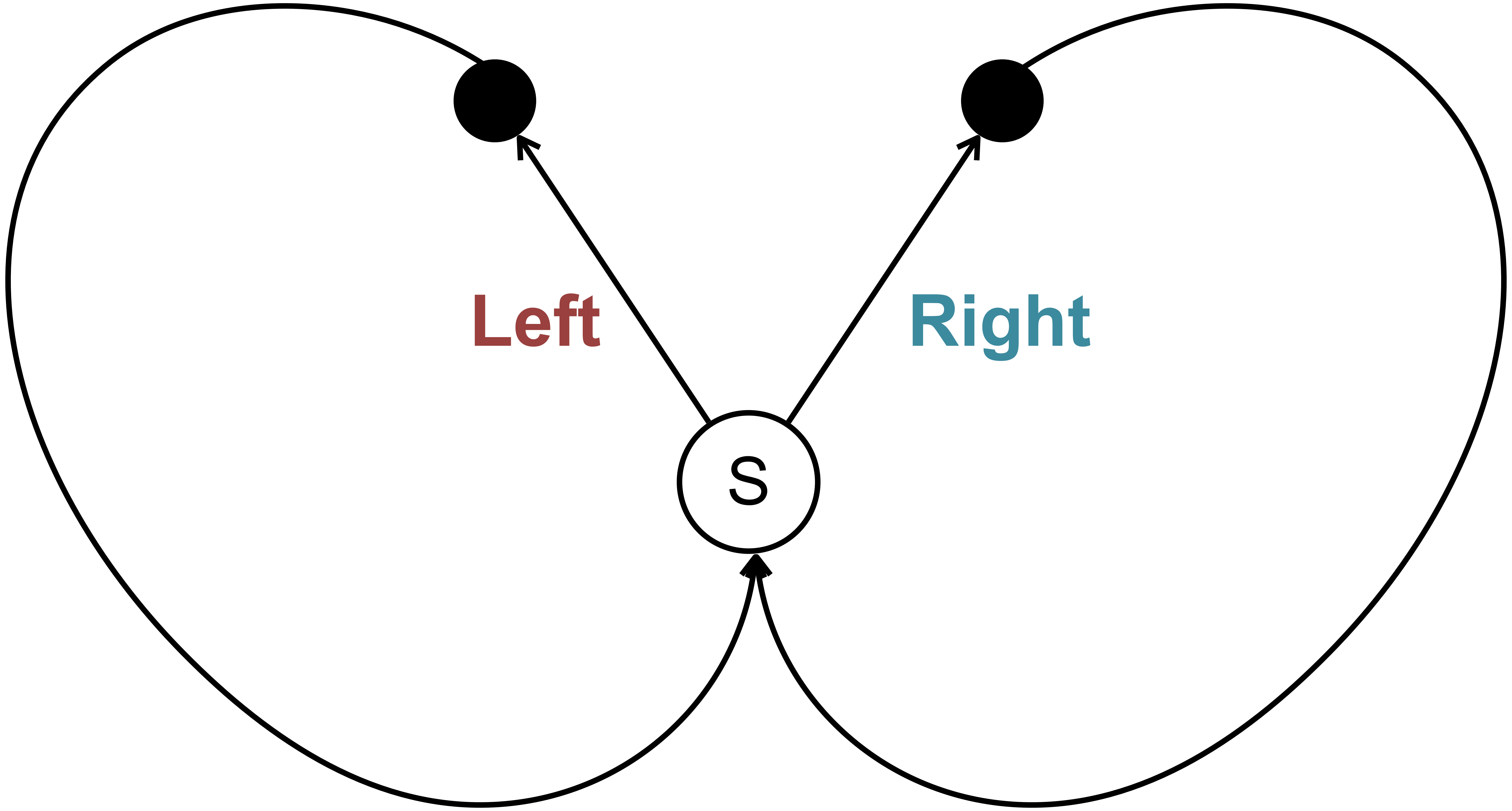


Valid and invalid policies



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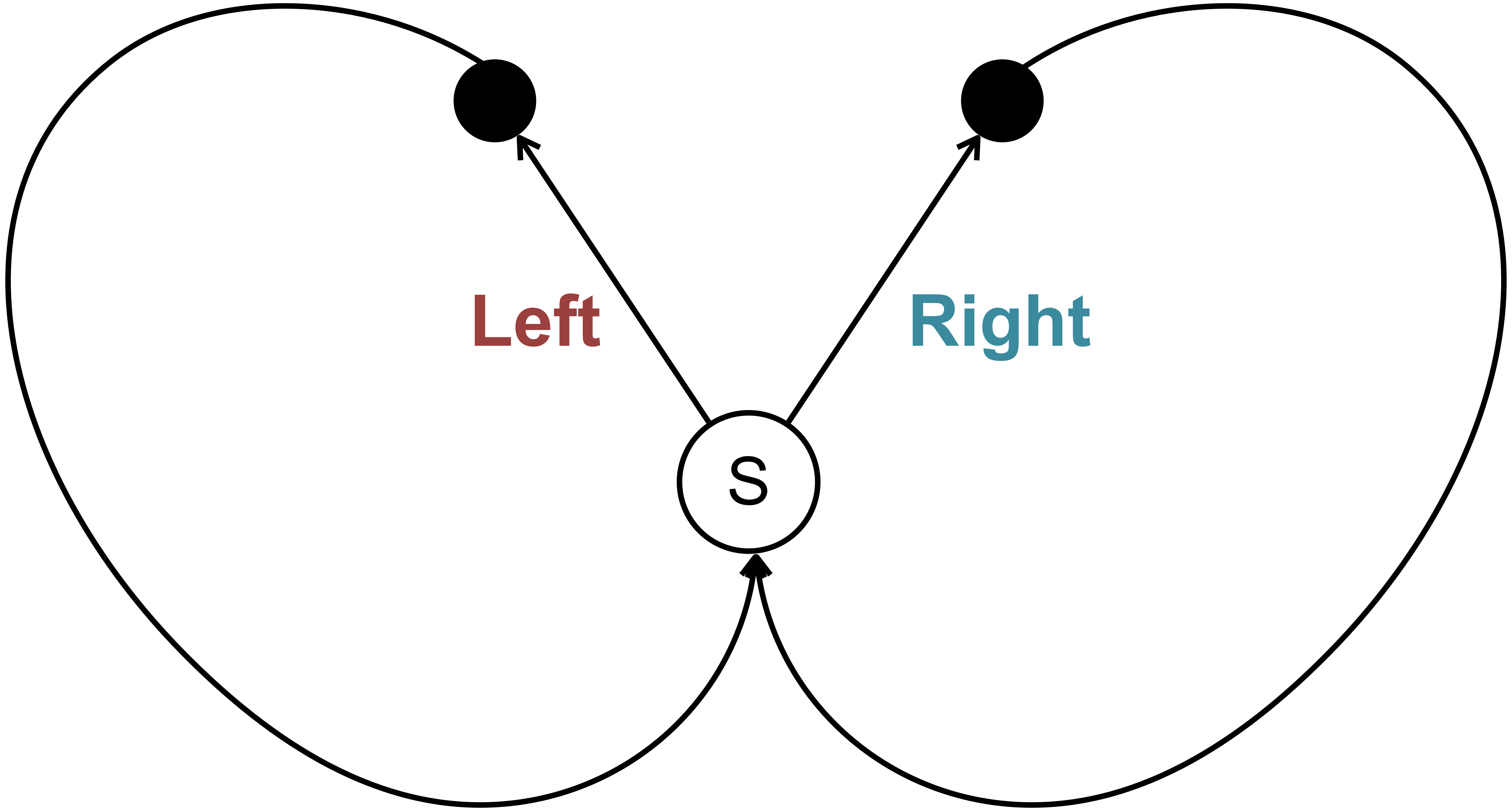
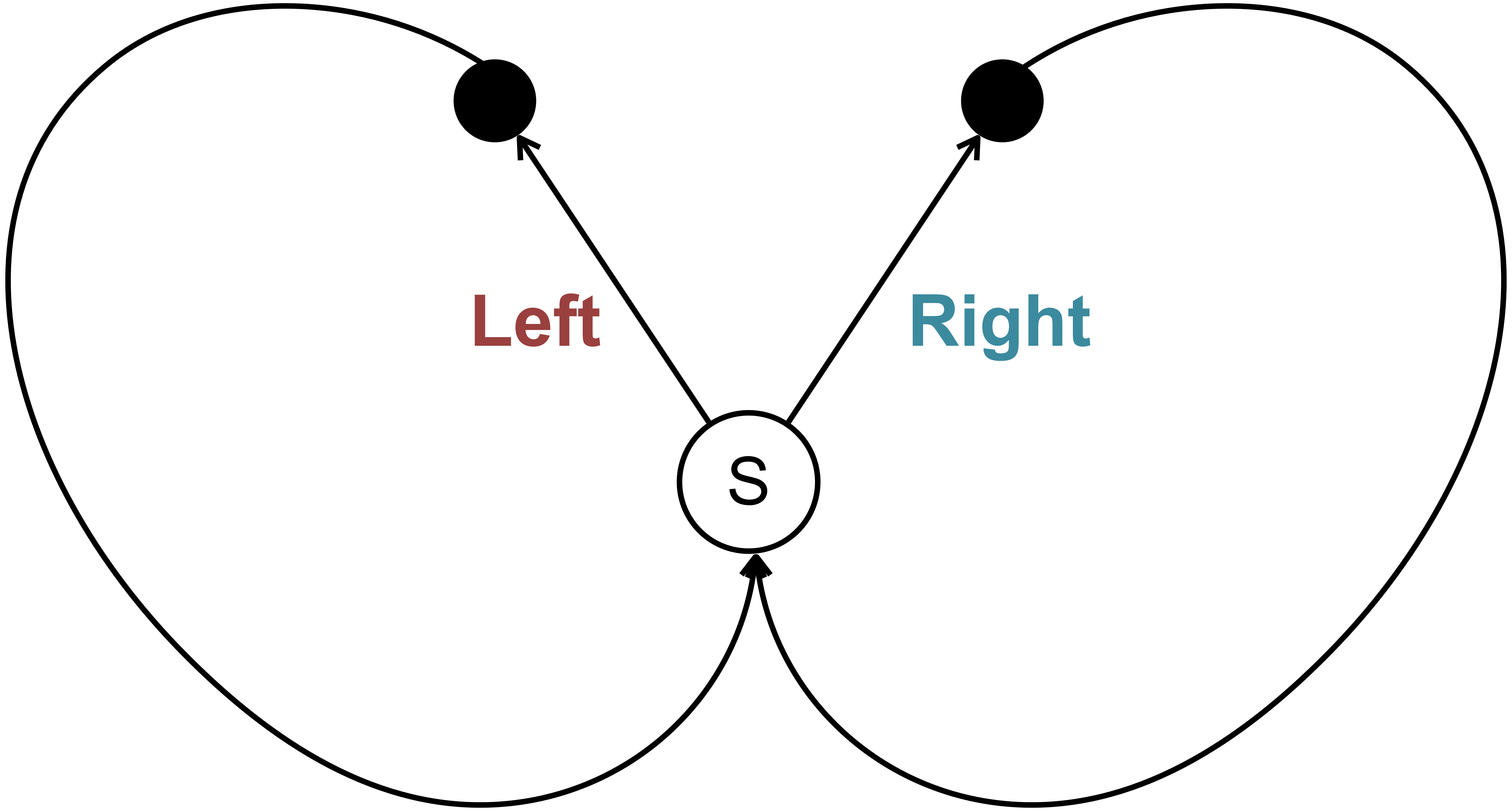


1: Left with 50% probability
and
Right with 50% probability

L L R L R L R R R ...

Valid and invalid policies



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2: Alternate Left and Right

L R L R L R L R L ...

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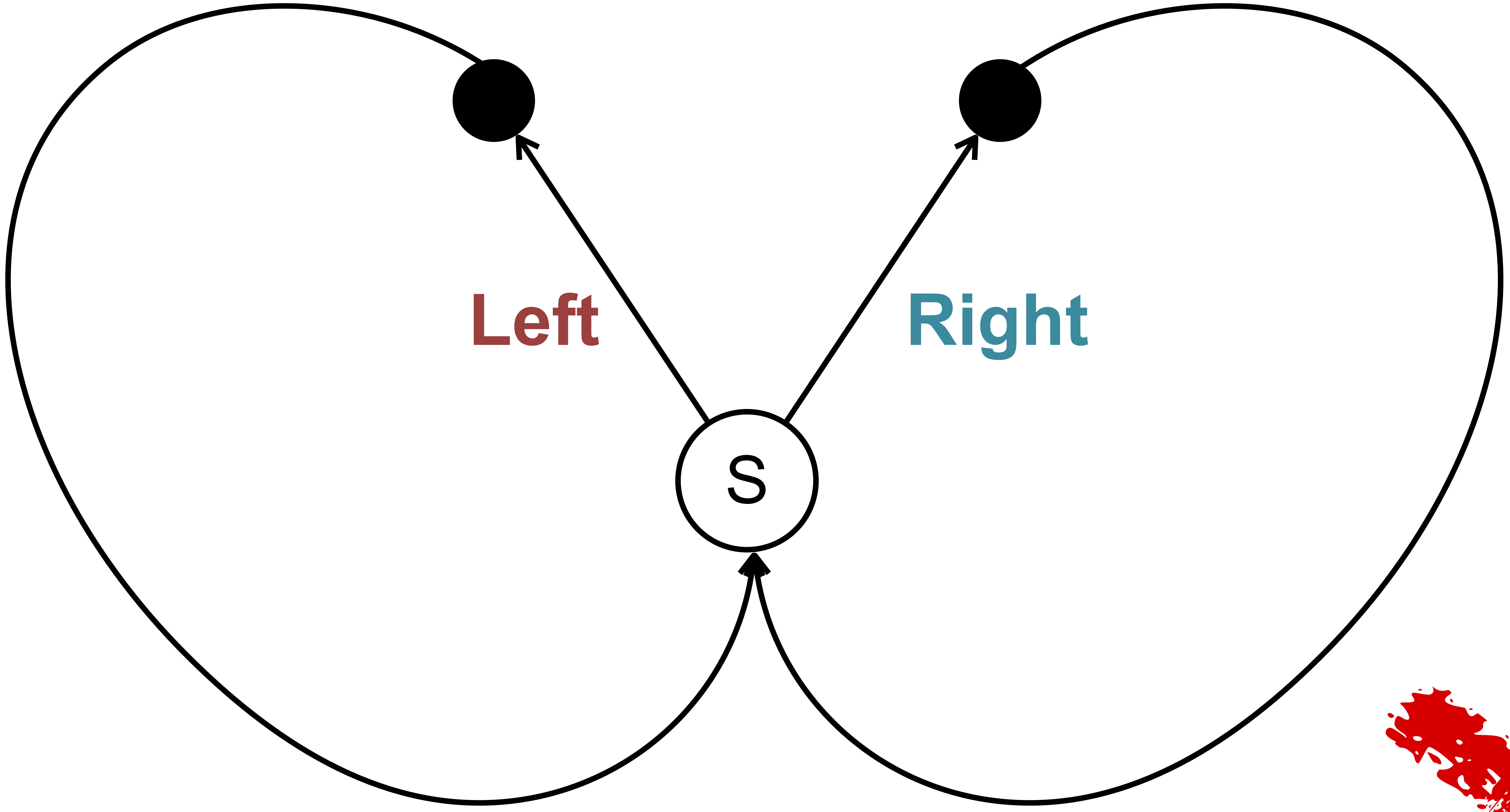

1: Left with 50% probability
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2: Alternate Left and Right

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Markov Property

Action-value functions

- An action-value function says how good it is to be in a state, take an action, and thereafter follow a policy:

$$q_\pi(s,a) \doteq \mathbb{E}_\pi\left[R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} \ldots \mid S_t = s, A_t = a\right]$$

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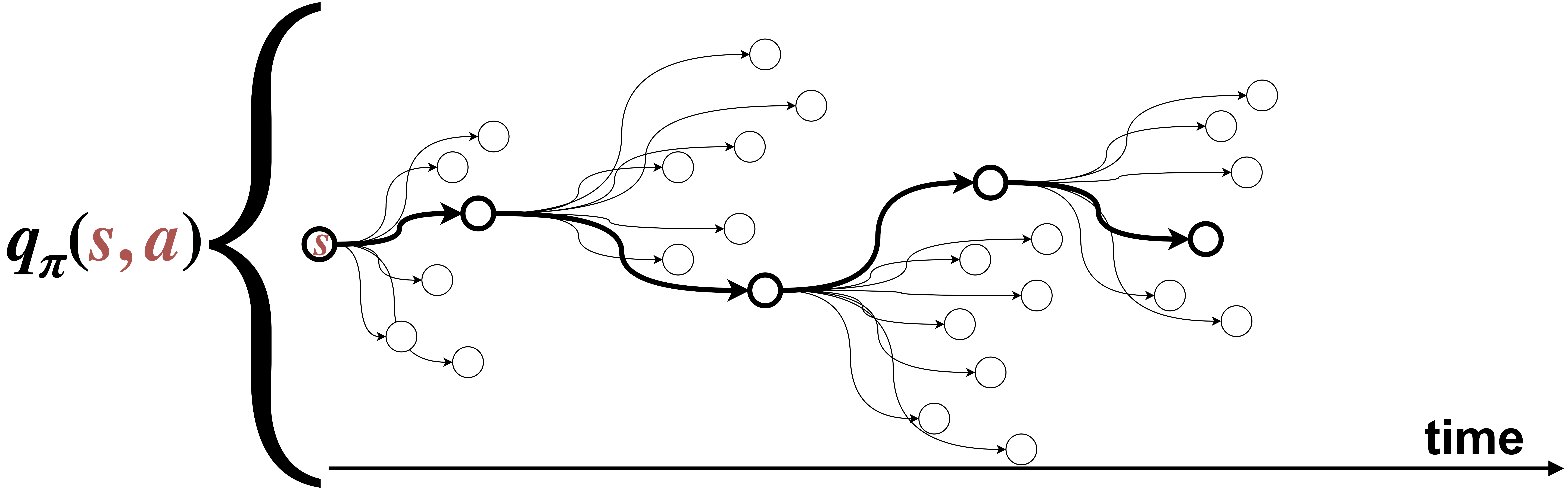


Optimal Polices

- A policy $\pi_\star$ is optimal if it maximizes the action-value function:

$$q_{\pi_\star}(s,a) \doteq \max_\pi q_\pi(s,a) = q_\star(s,a) \qquad \forall\, s \in \mathcal{S},\, a \in \mathcal{A}(s)$$

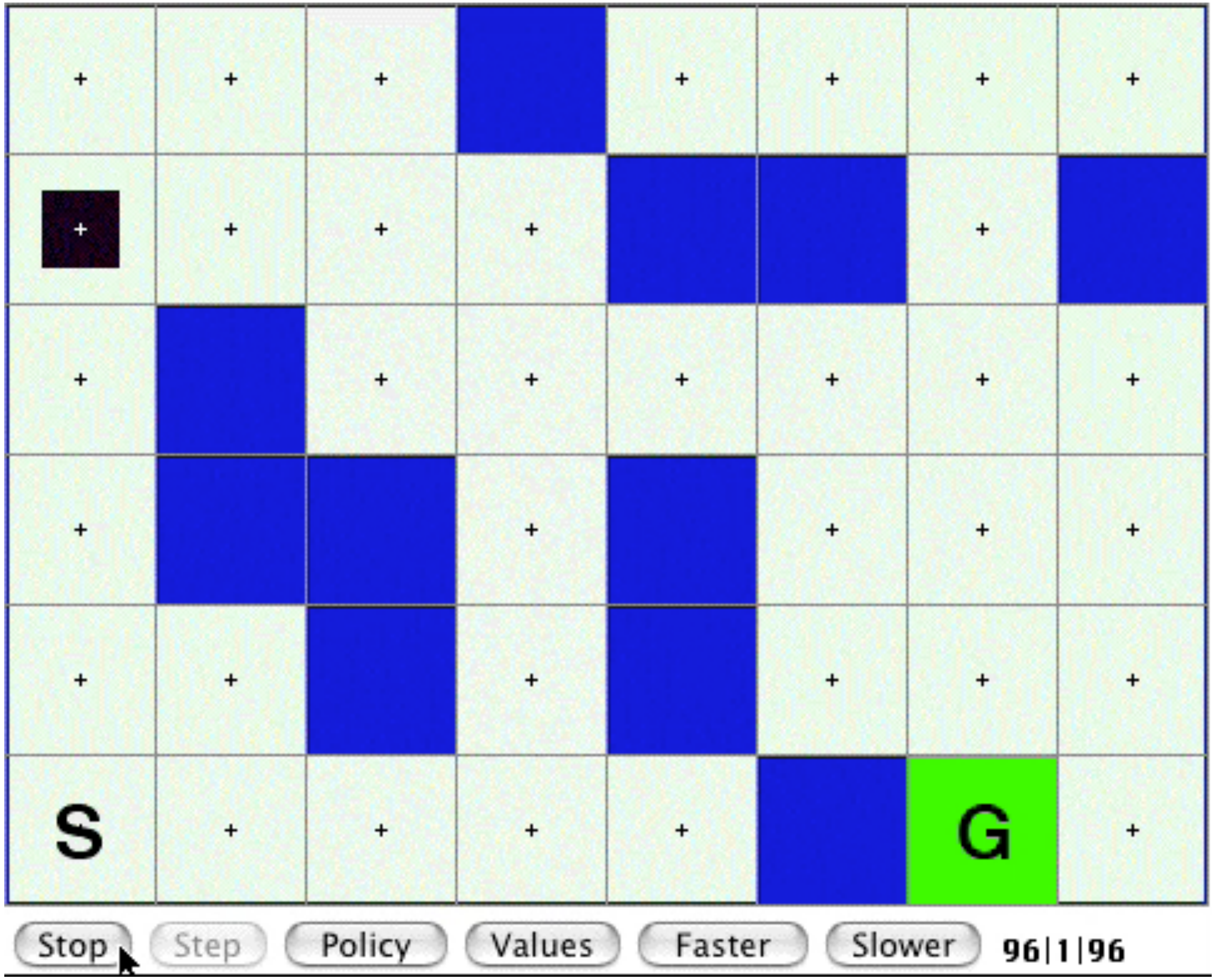
- Thus all optimal policies share the same optimal value function
- Given the optimal value function, it is easy to act optimally:

$$\pi_\star(s) = \arg\max_a q_\star(s,a) \qquad \text{“greedification”}$$

  - we say that the optimal policy is greedy with respect to the optimal value function

- There is always at least one deterministic optimal policy

GridWorld Example



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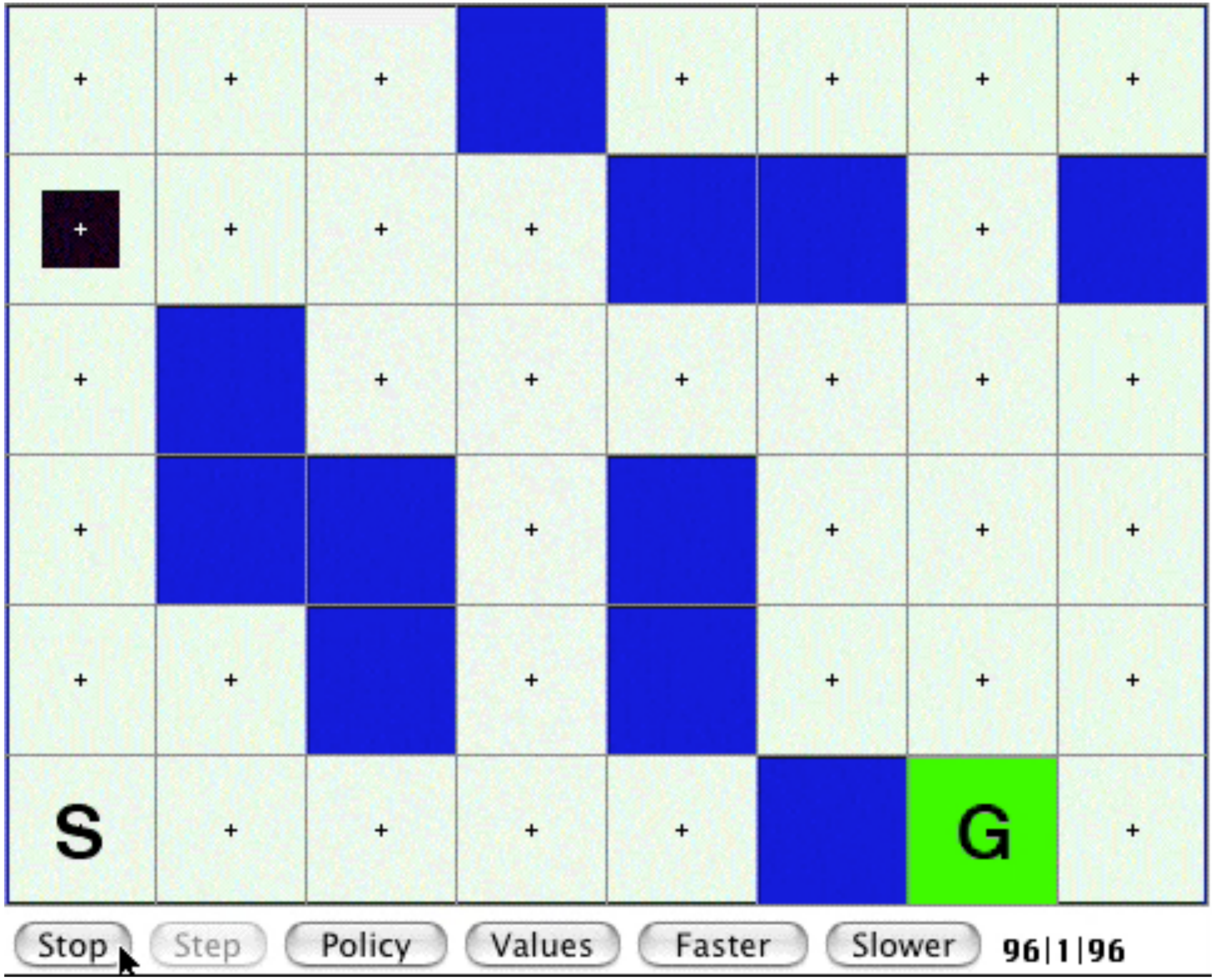
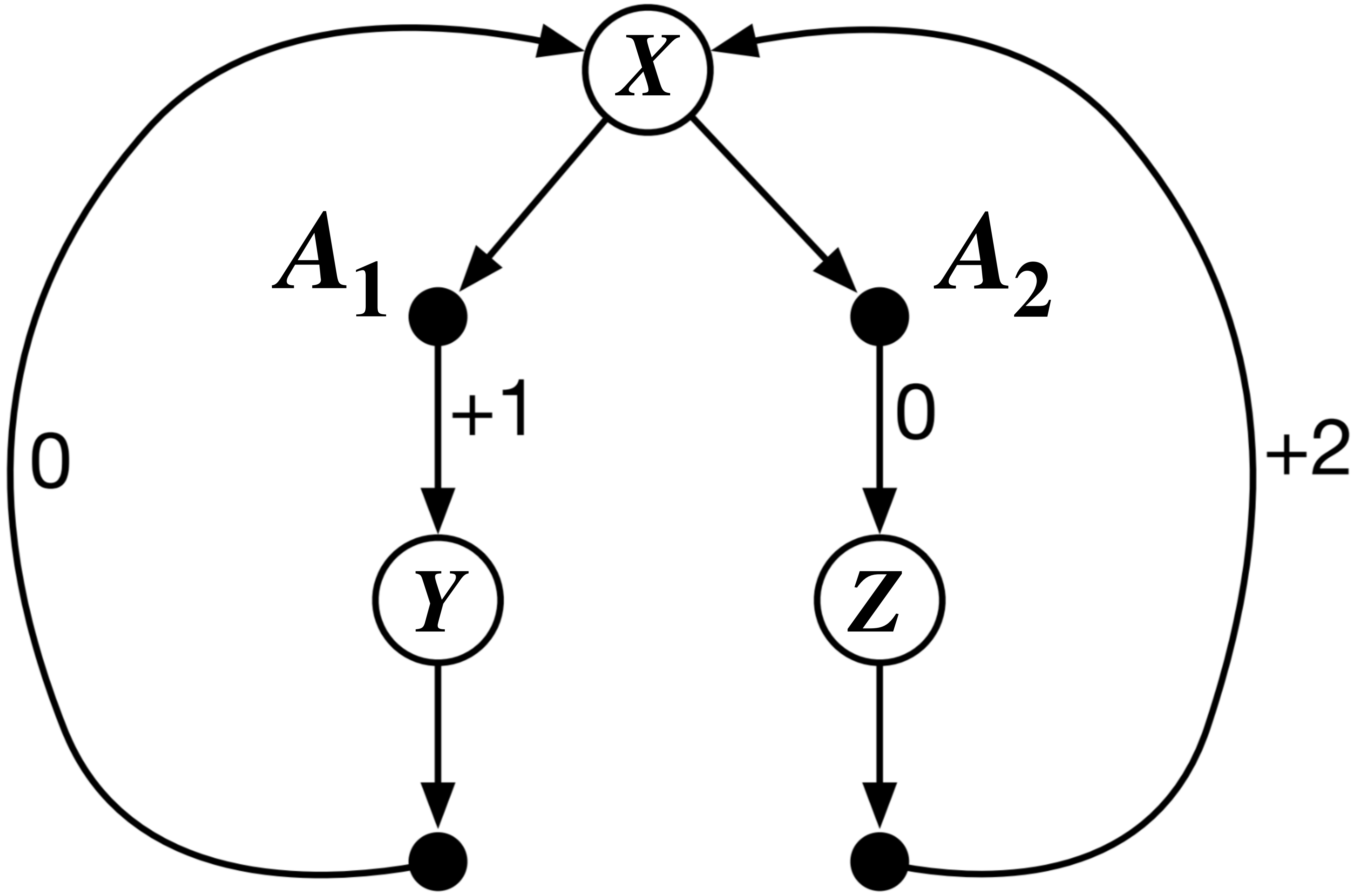


Exercise: what's optimal?



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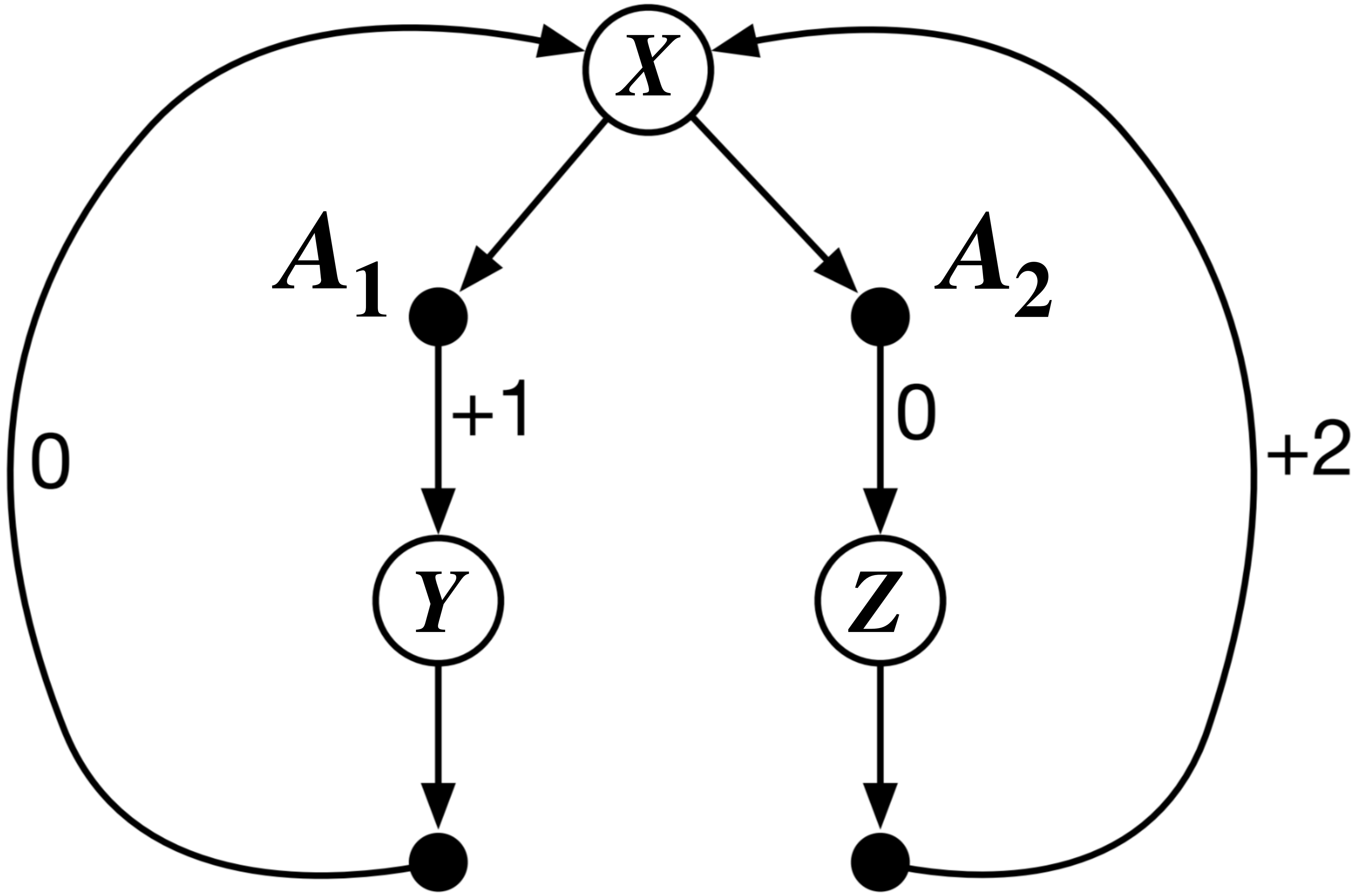
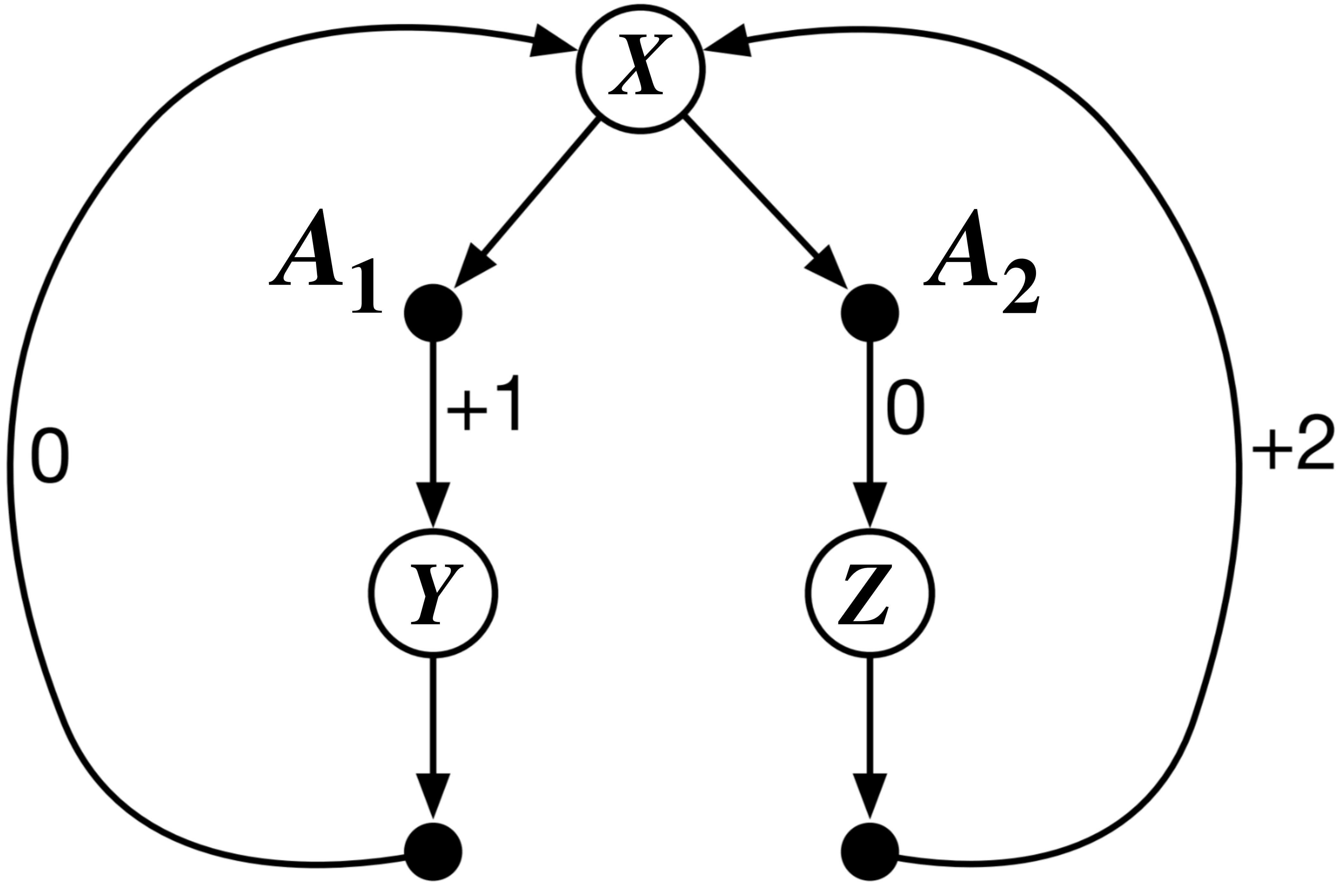


$\pi_1(X) = A_1$ $\pi_2(X) = A_2$

Exercise: what's optimal?



$\gamma = 0$

$\pi_1(X) = A_1$ $\pi_2(X) = A_2$

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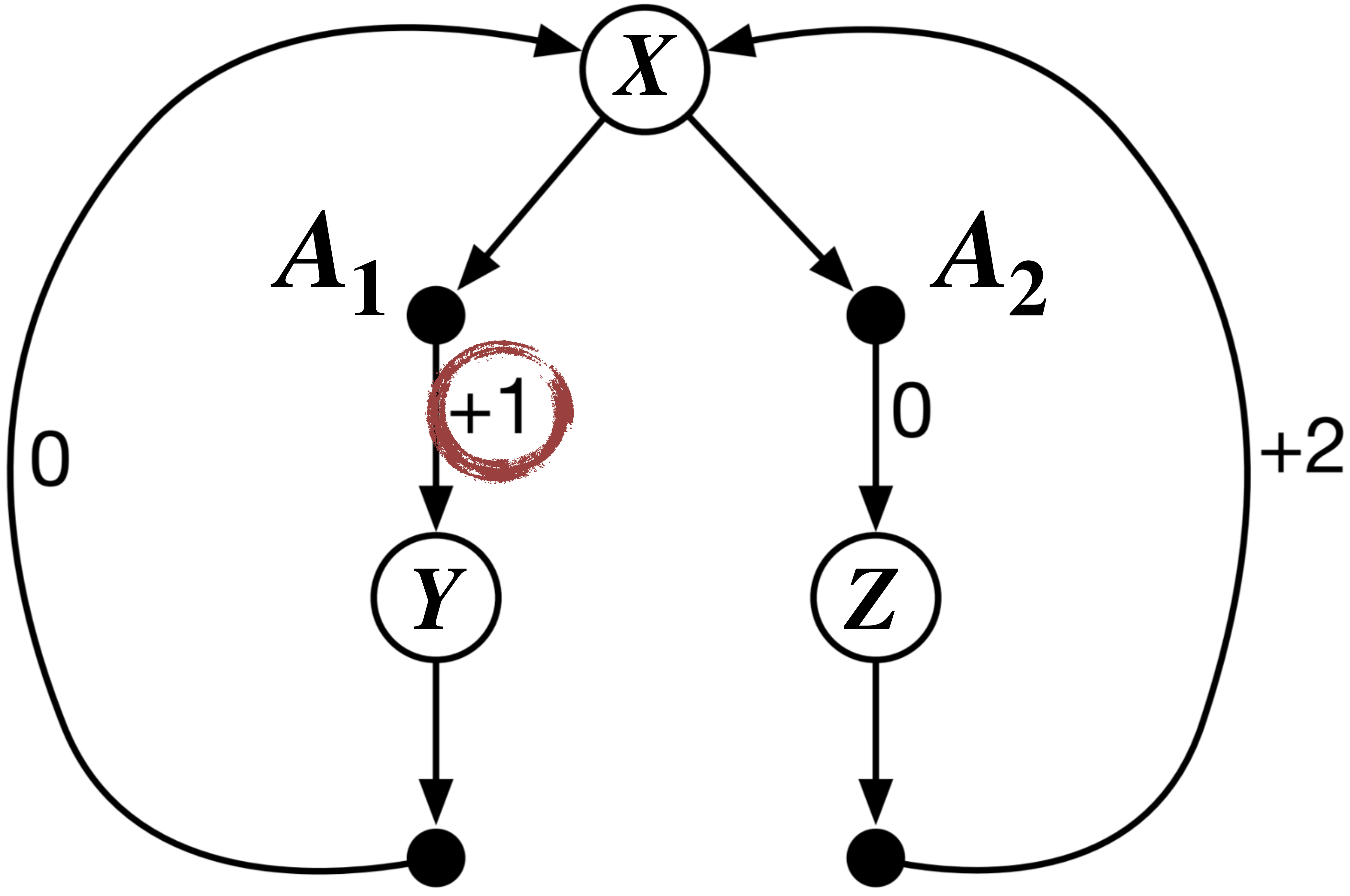

$\gamma = 0$

$v_{\pi_1}(X) = 1$

$\pi_1(X) = A_1$ $\quad$ $\pi_2(X) = A_2$

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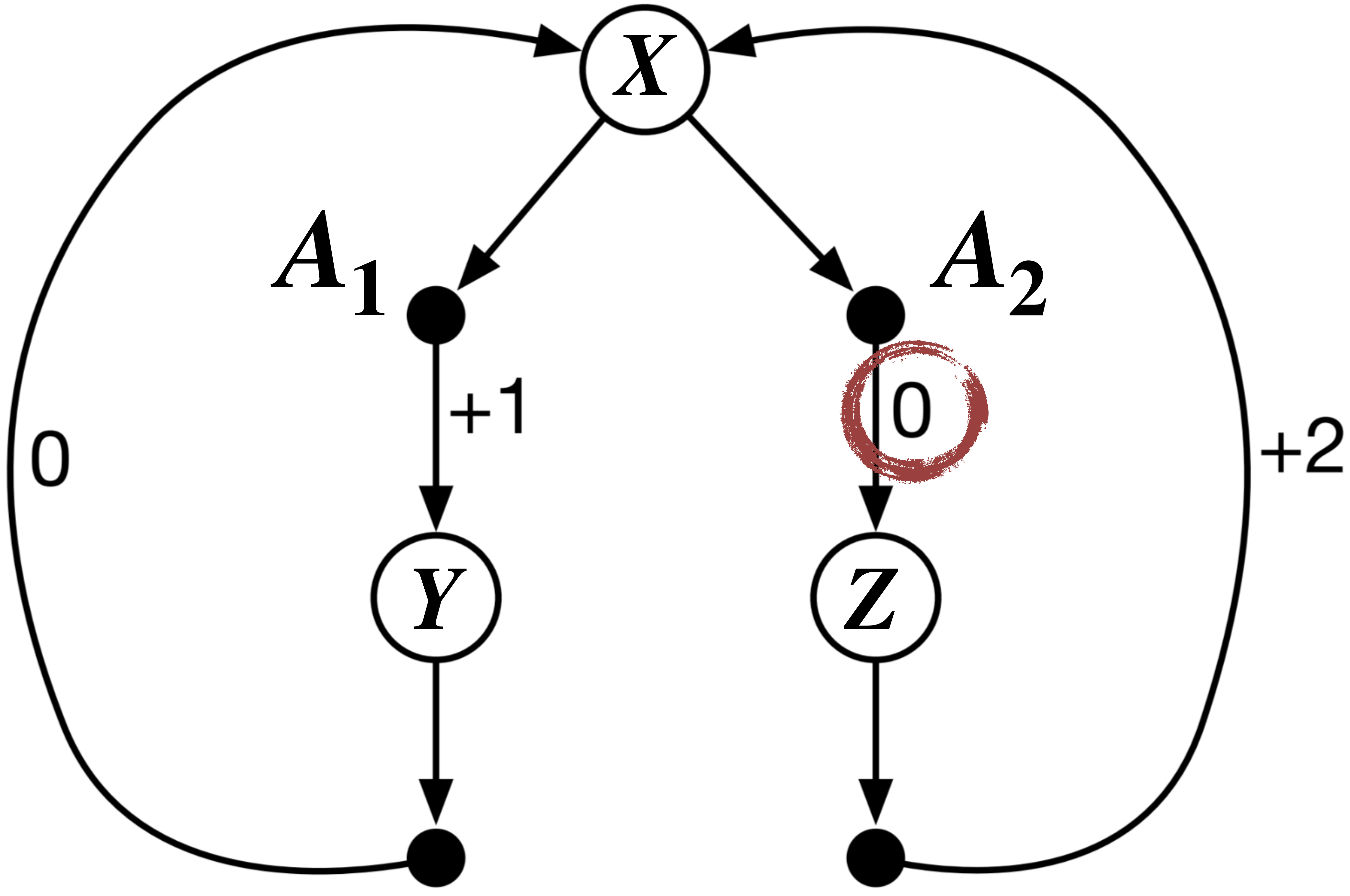

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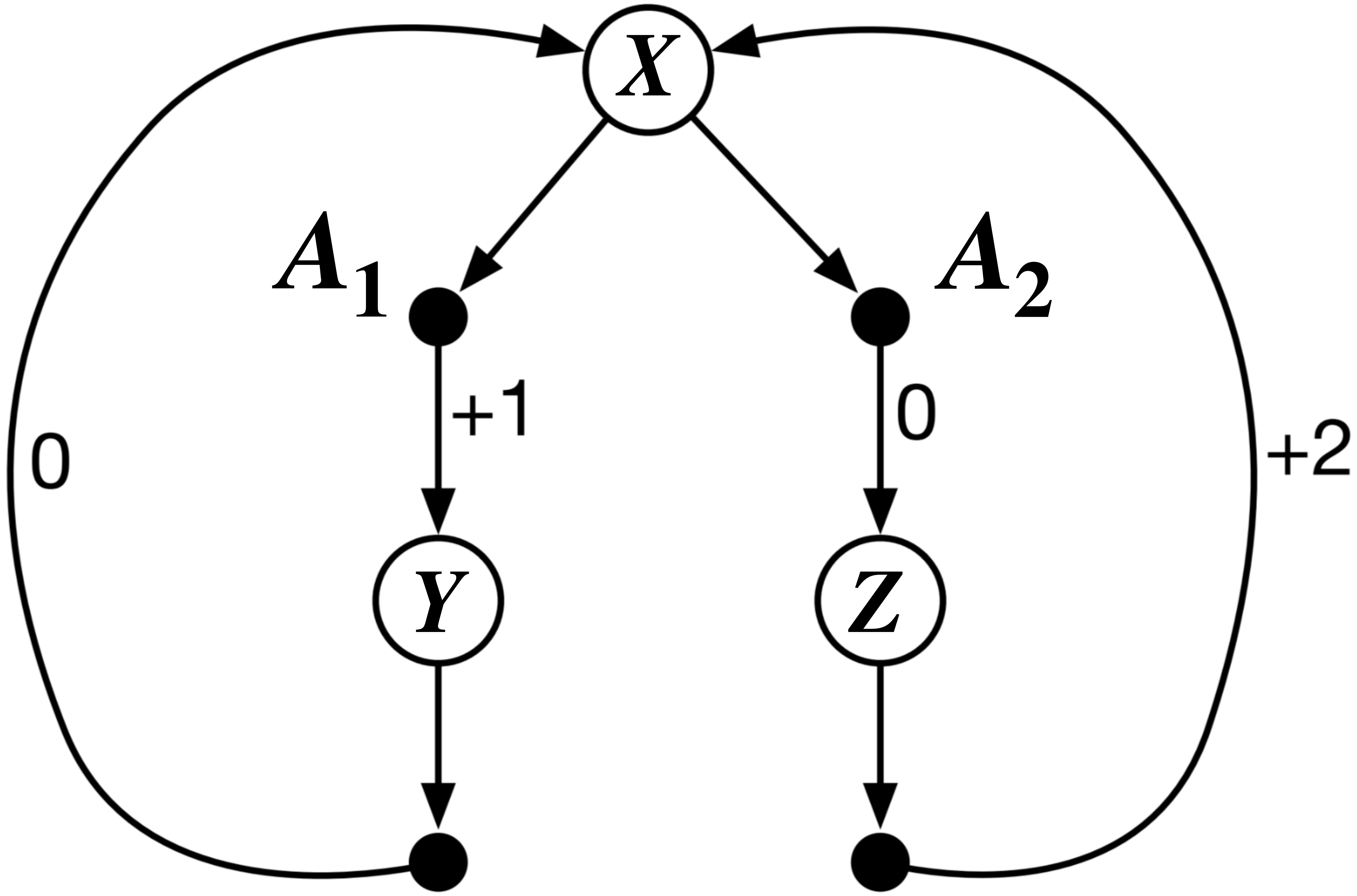

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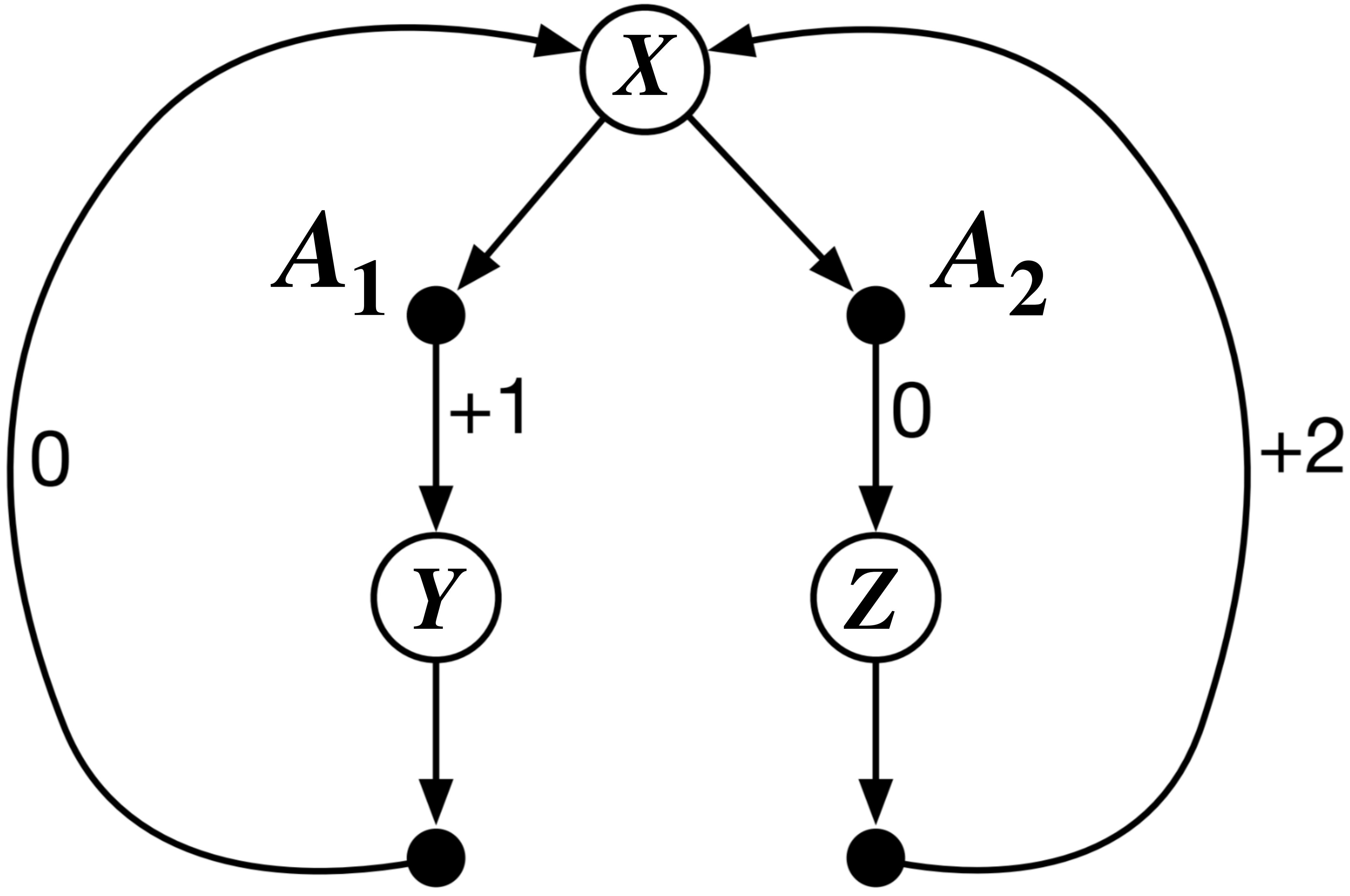

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$\gamma = 0.9$

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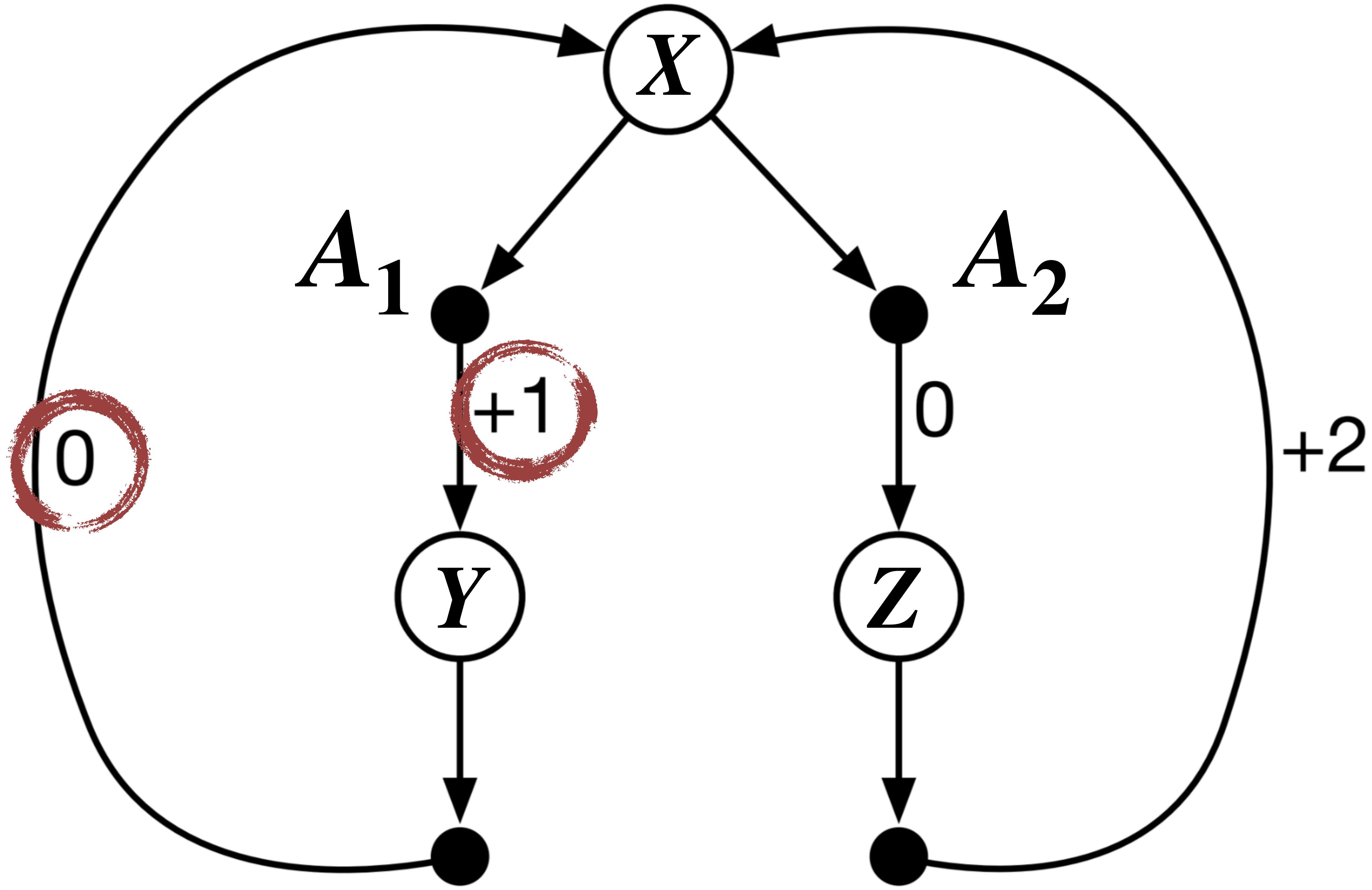


$\pi_1(X) = A_1 \quad \pi_2(X) = A_2$

$\gamma = 0$

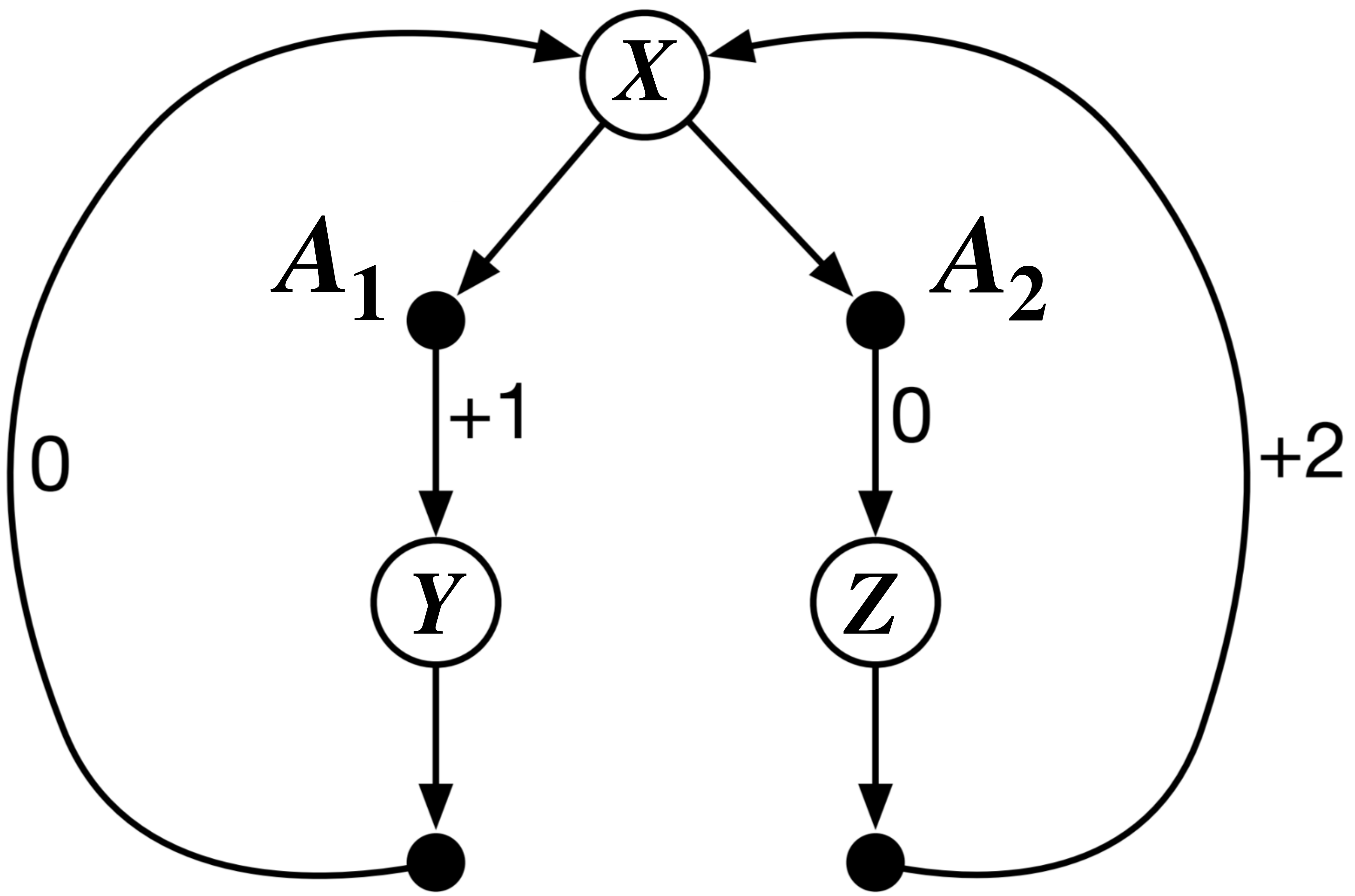
$v_{\pi_1}(X) = 1$ ✓

$v_{\pi_2}(X) = 0$

$\gamma = 0.9$

$v_{\pi_1}(X) = 1 + 0.9 * 0 + (0.9)^2 * 1 + \ldots$

Exercise: what's optimal?



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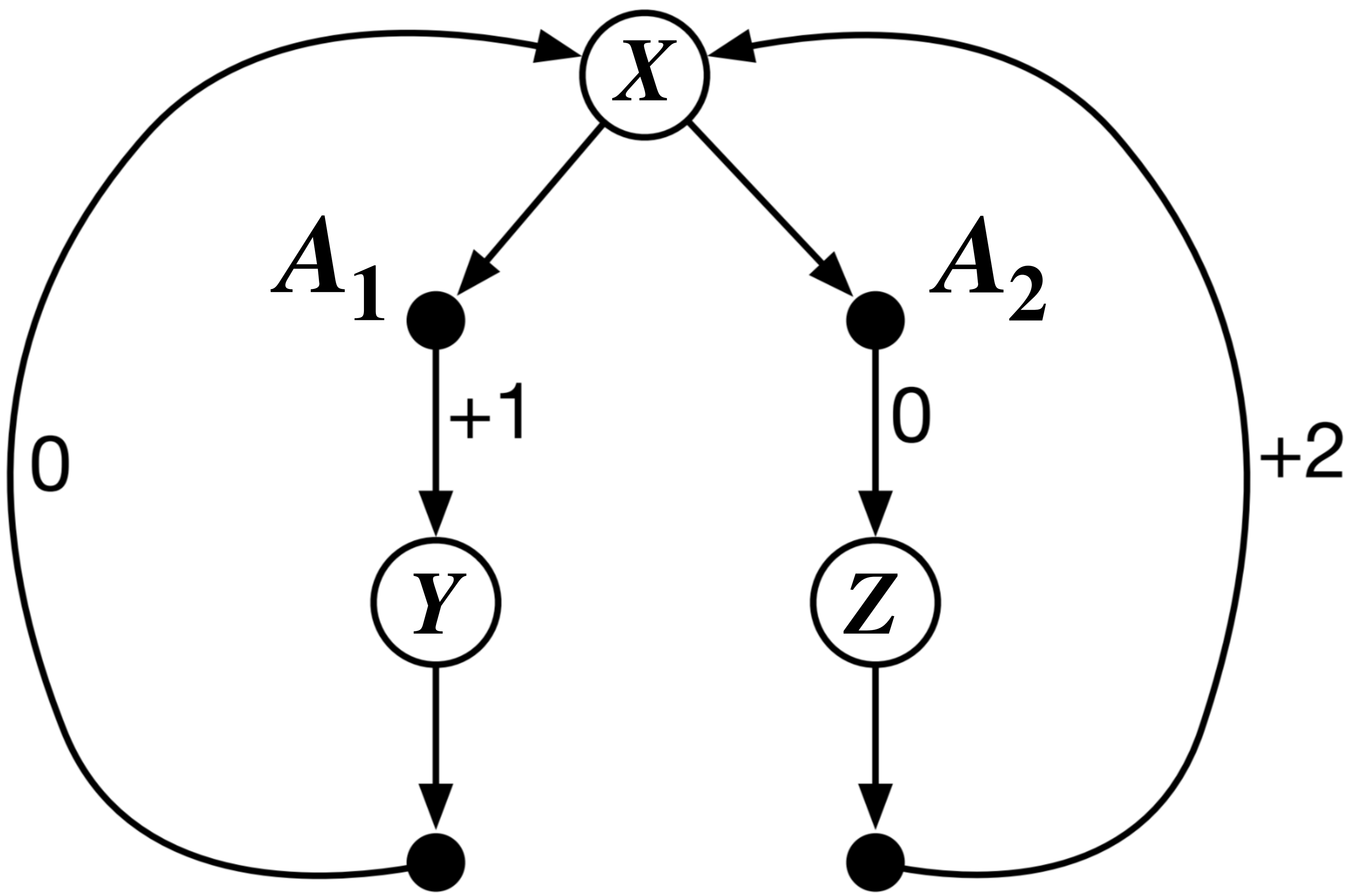
$$v_{\pi_1}(X) = \sum_{k=0}^{\infty} (0.9)^{2k}$$

Exercise: what's optimal?



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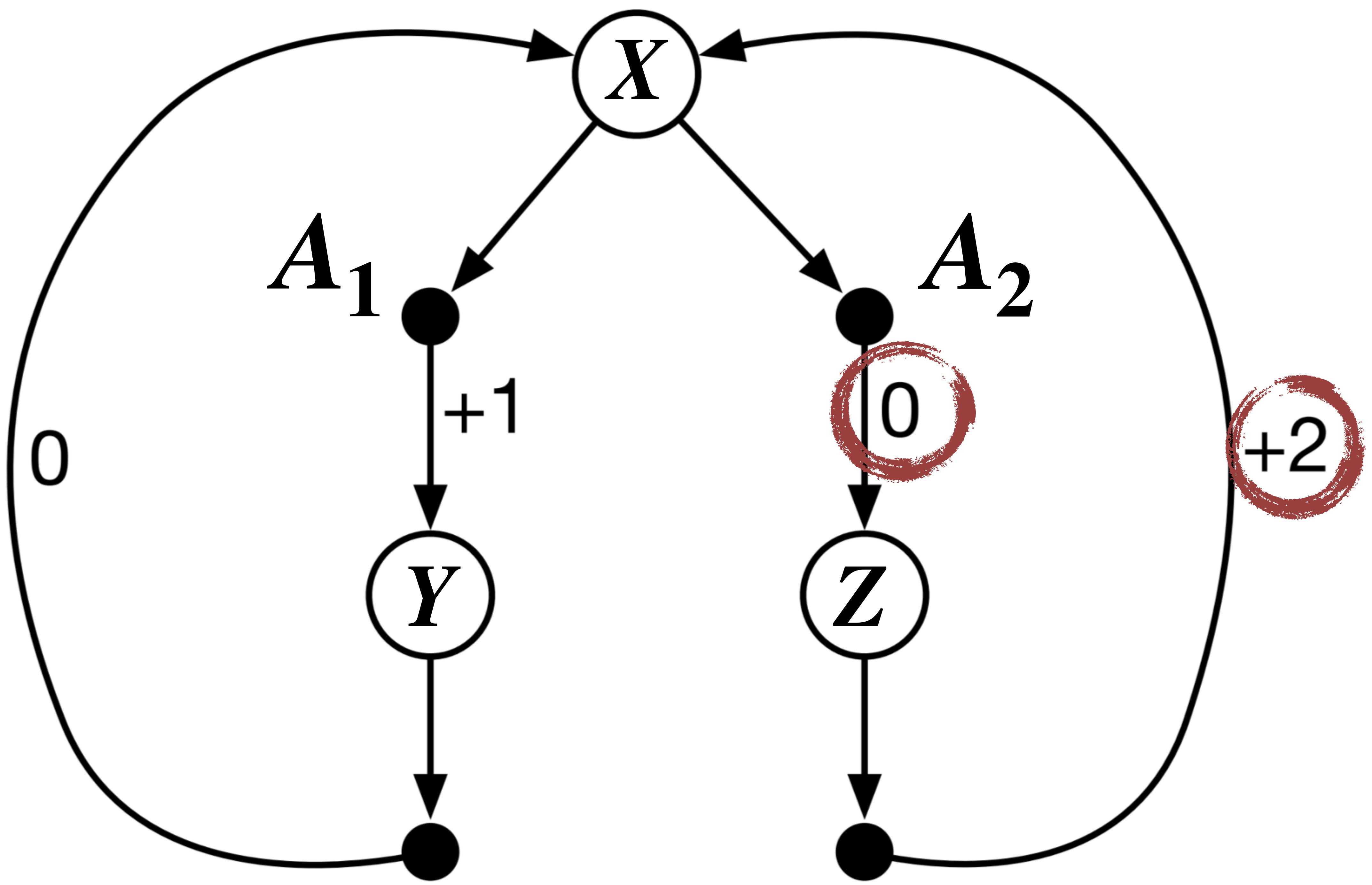
$$v_{\pi_1}(X) = \sum_{k=0}^{\infty} (0.9)^{2k} = \frac{1}{1 - 0.9^2} \approx 5.3$$

Exercise: what's optimal?



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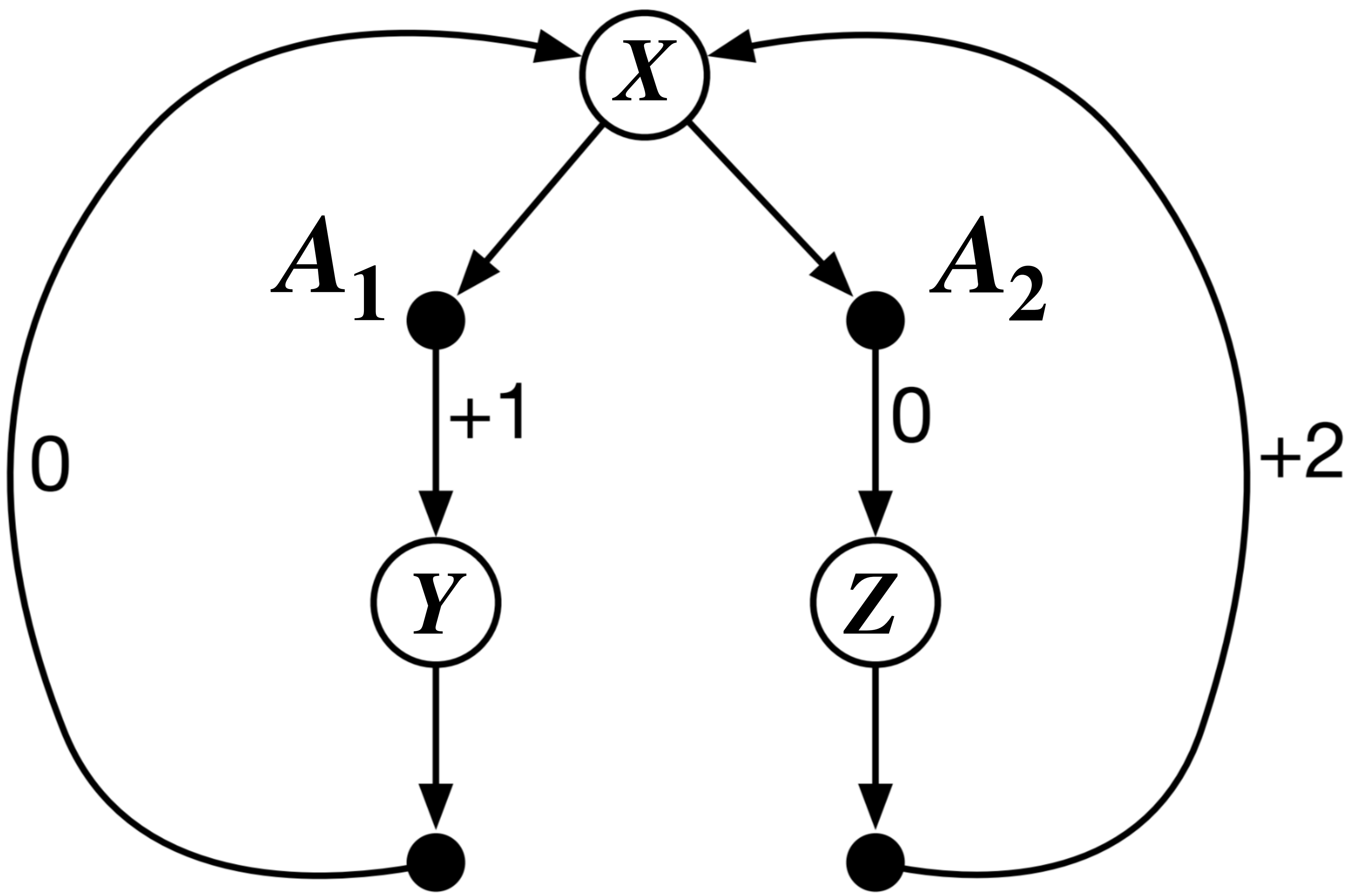

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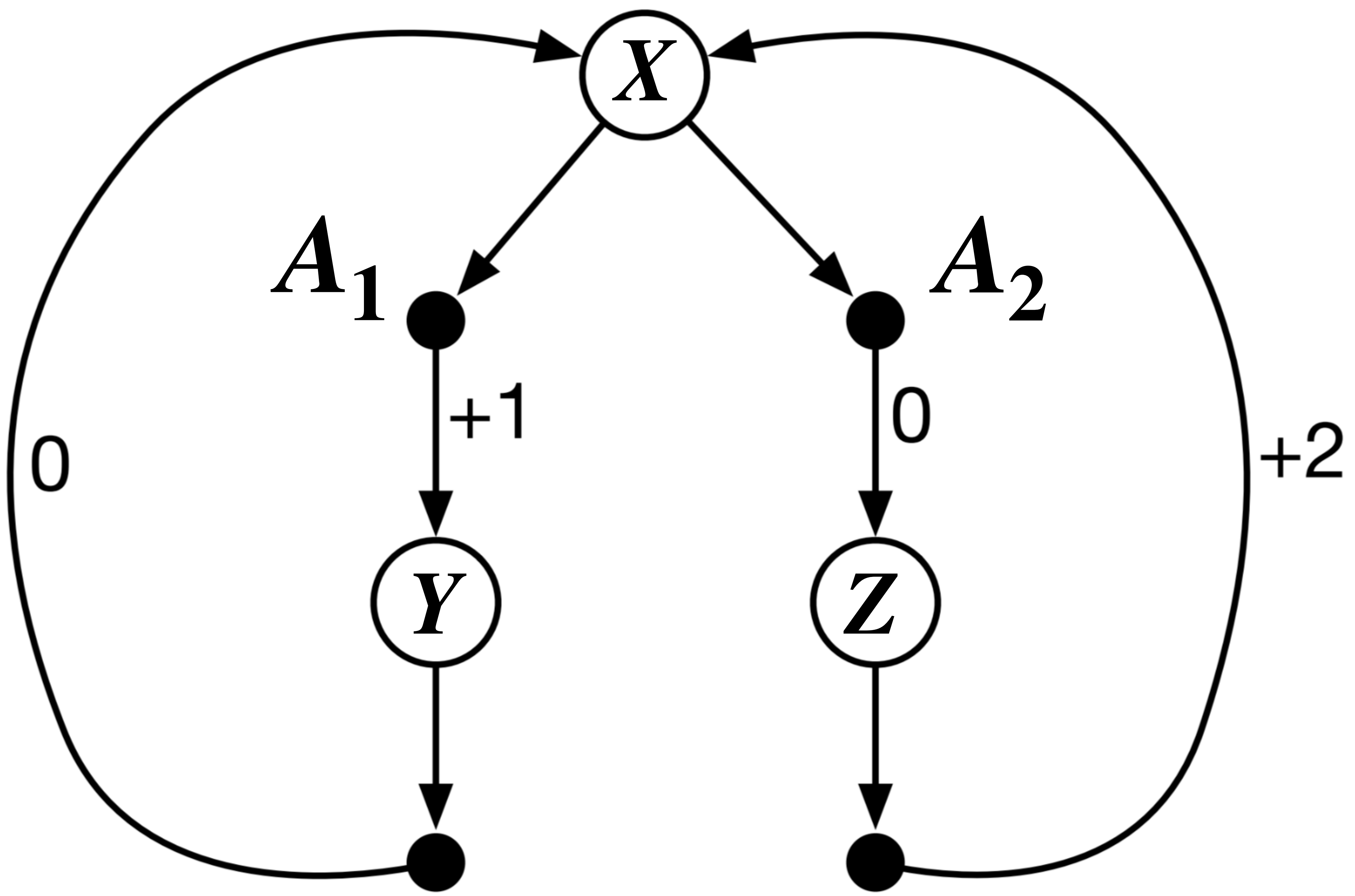

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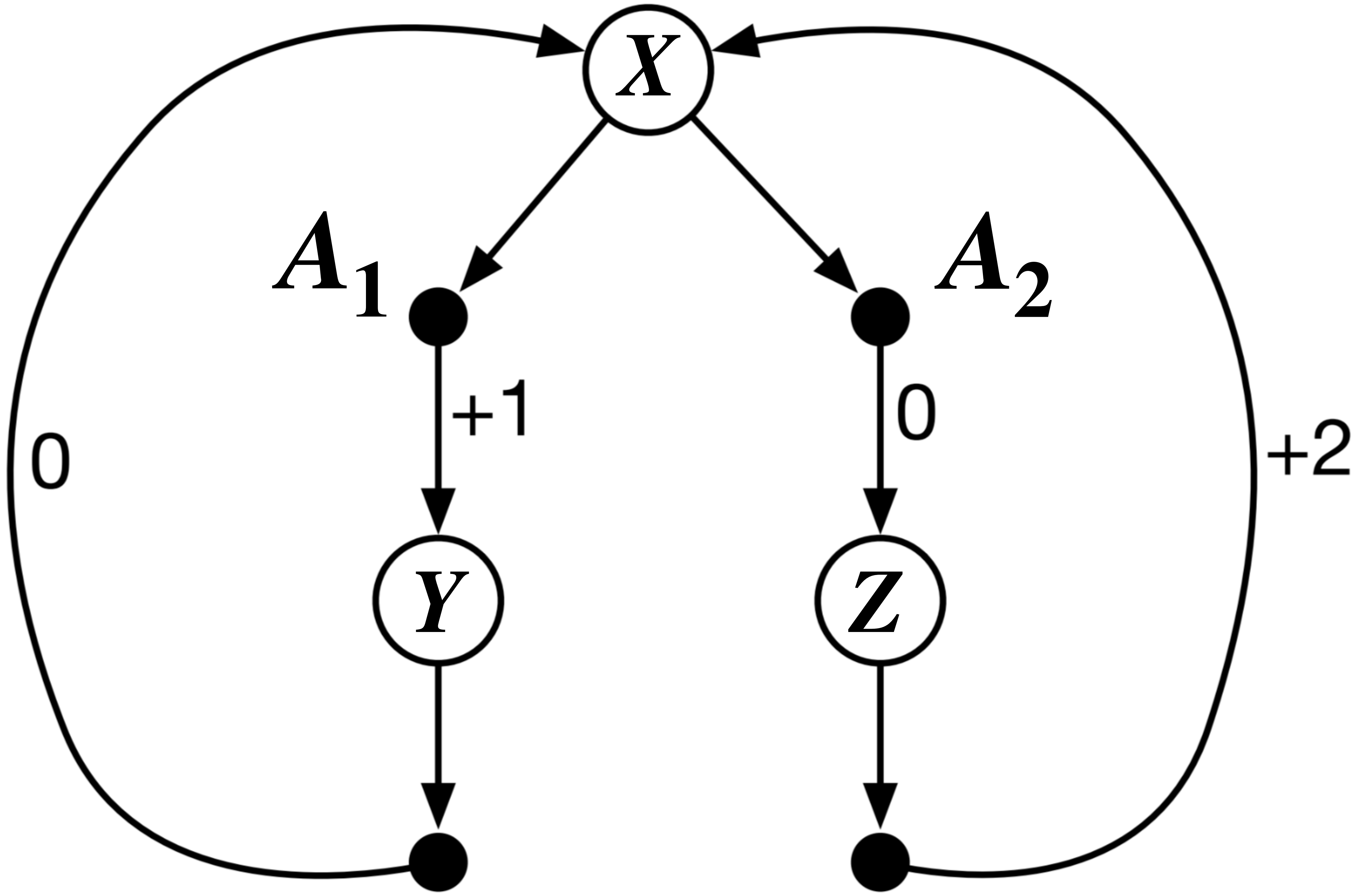

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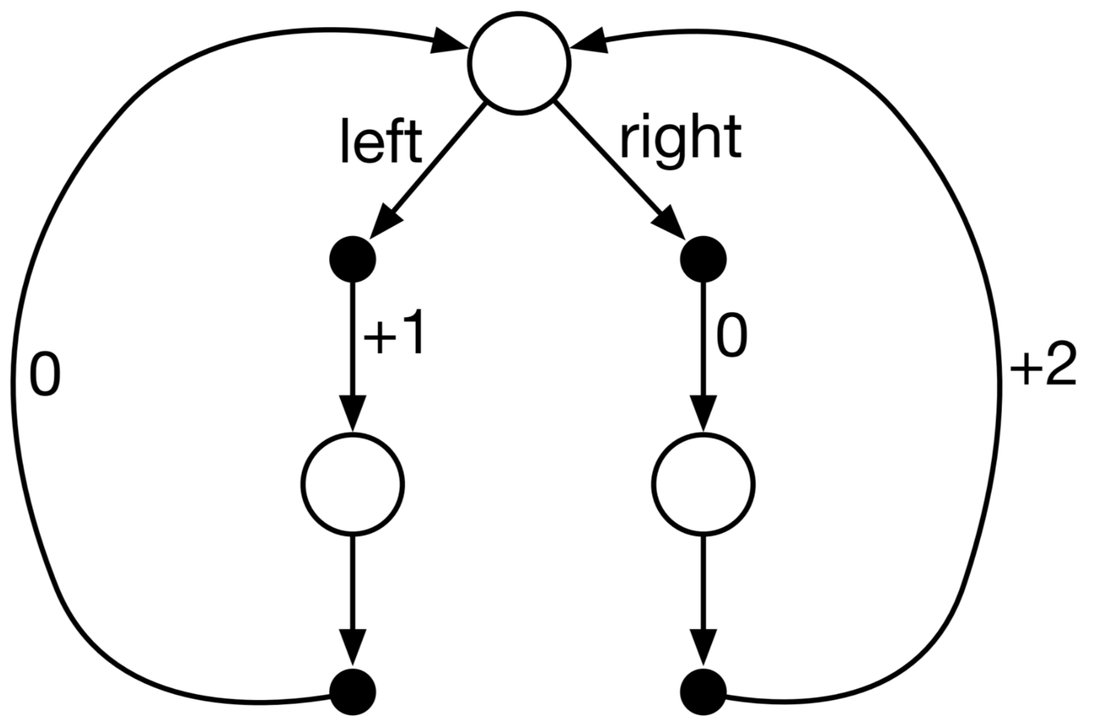

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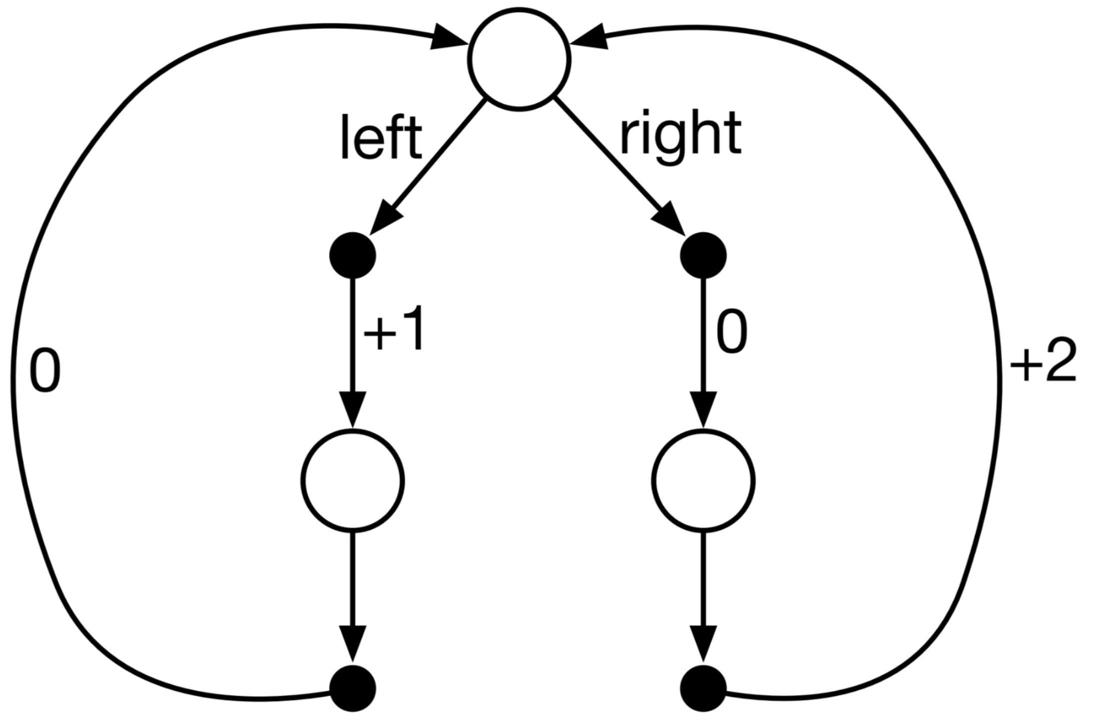

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$|\mathscr{A}|^{|\mathscr{S}|}$ Deterministic Policies → RL methods based on Bellman Equations

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- ☑ Evaluative feedback (reward)
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Q-learning (off-policy TD control) for estimating $\pi \approx \pi_*$

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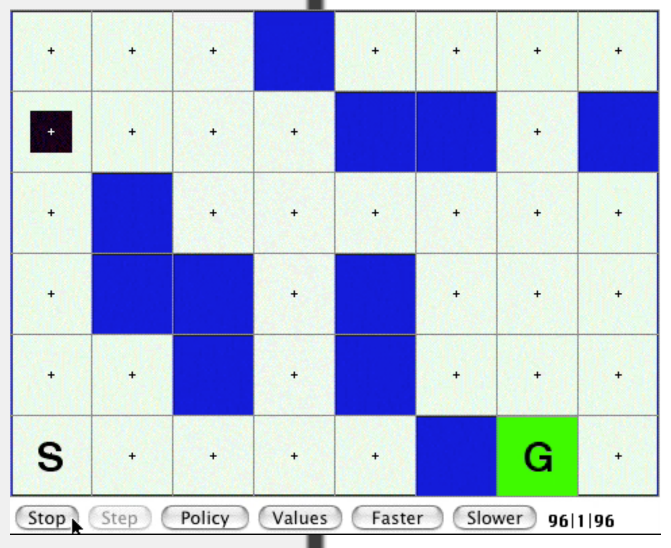

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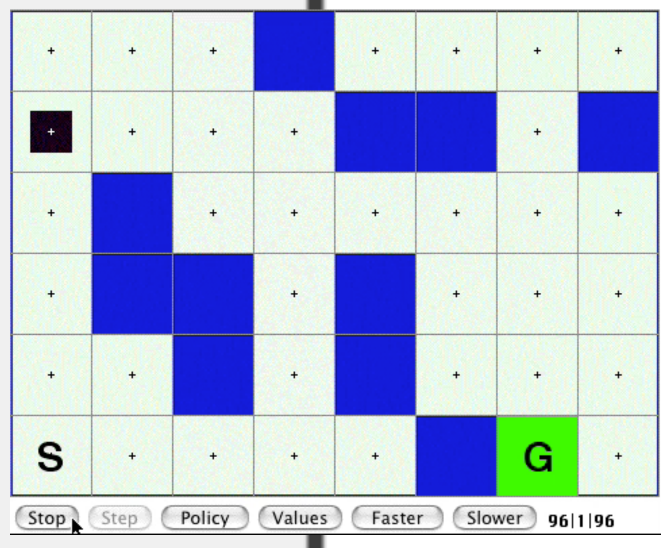

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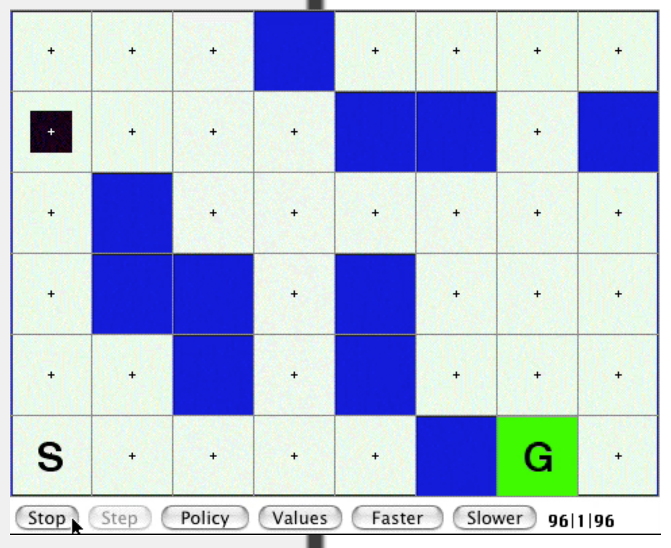

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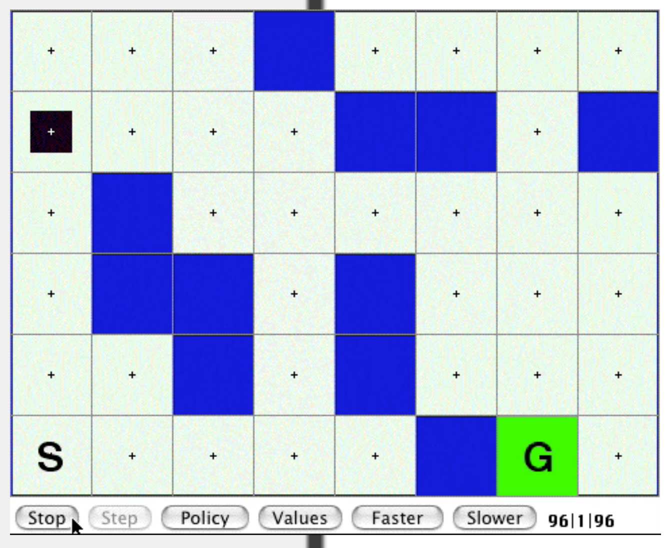

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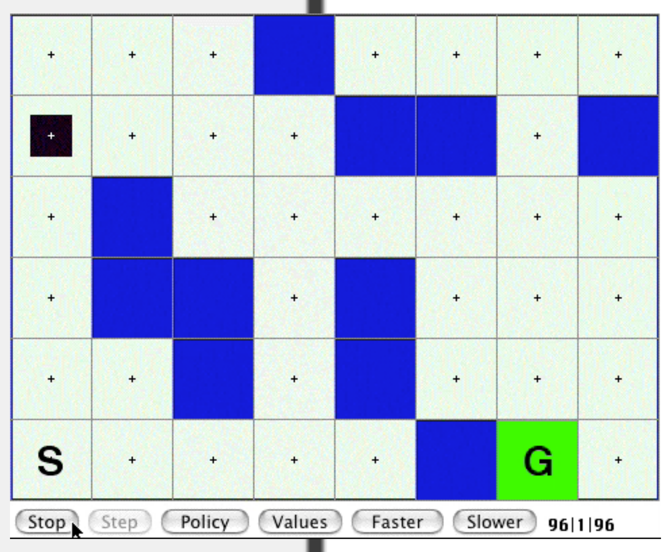

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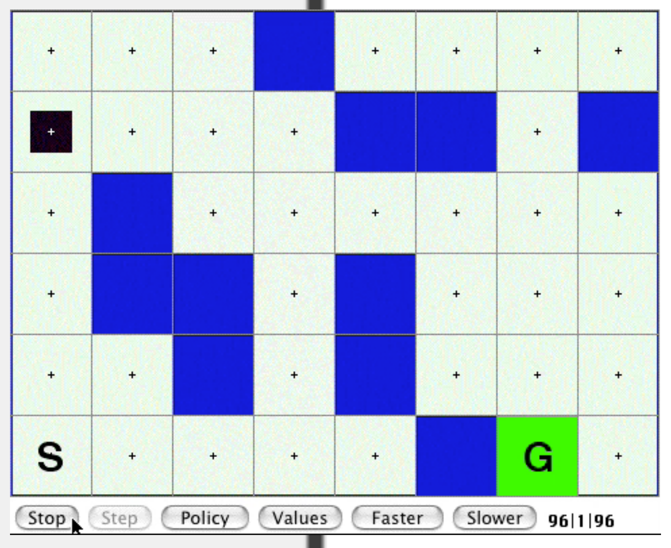

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Q converges to $q_\star$

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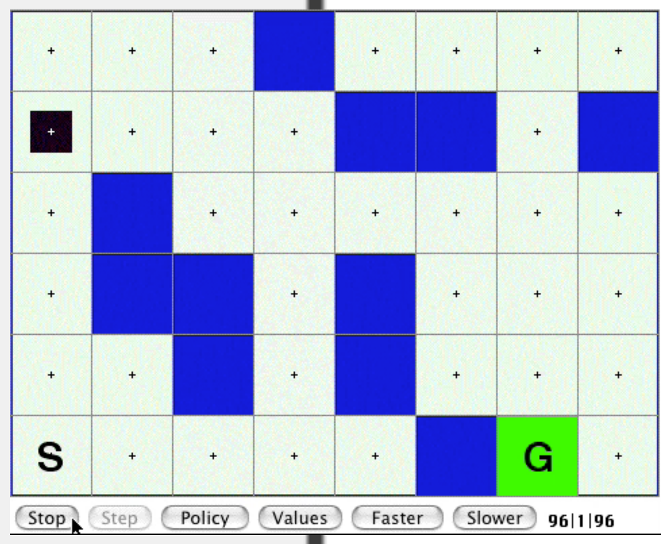

- Q-learning converges (Watkins & Dayan 1992) — learning long-term optimal behavior without any **model** of the environment, for arbitrary MDPs!

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- You might think we need a complete trajectory of rewards to estimate values

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- **Many** algorithms in RL, like Q-learning, can be seen as approximately solving the Bellman Equation with samples from the environment (model-free)

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- You must balance exploiting (picking what you think is the best), and exploring (refining your estimates)

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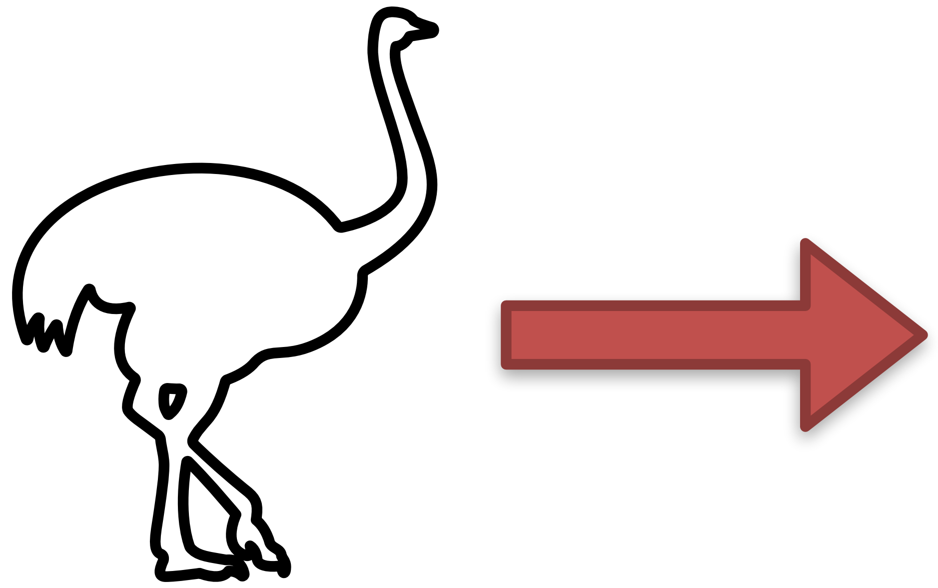

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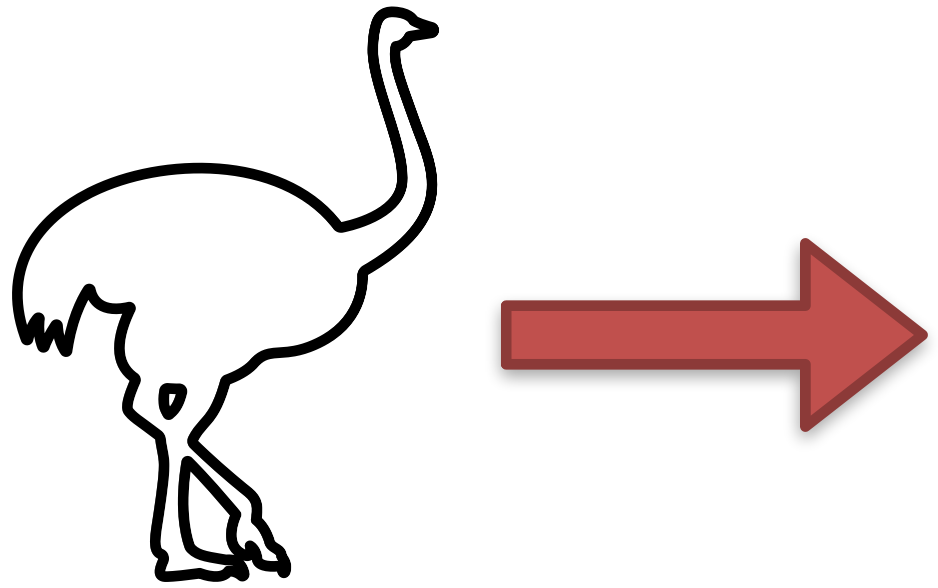

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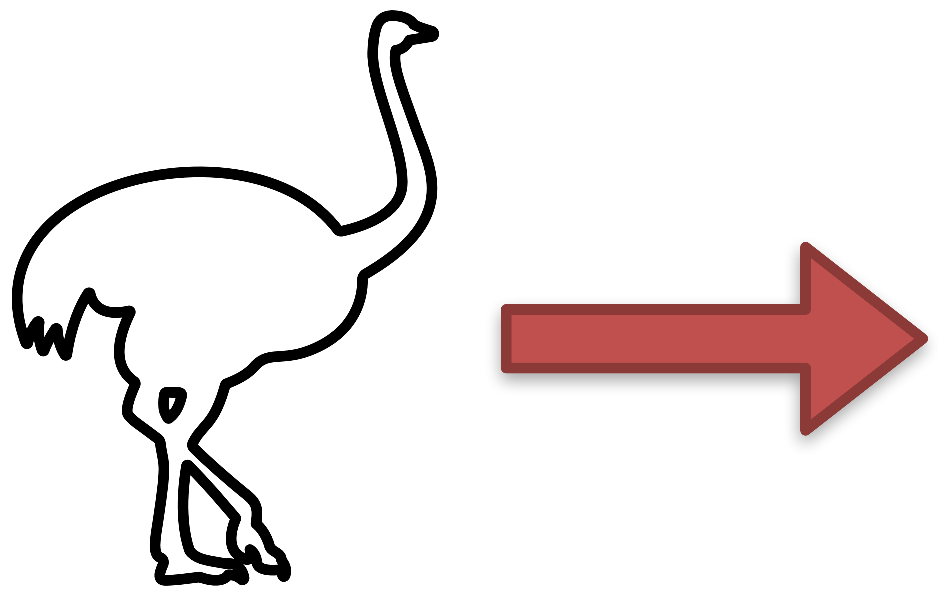

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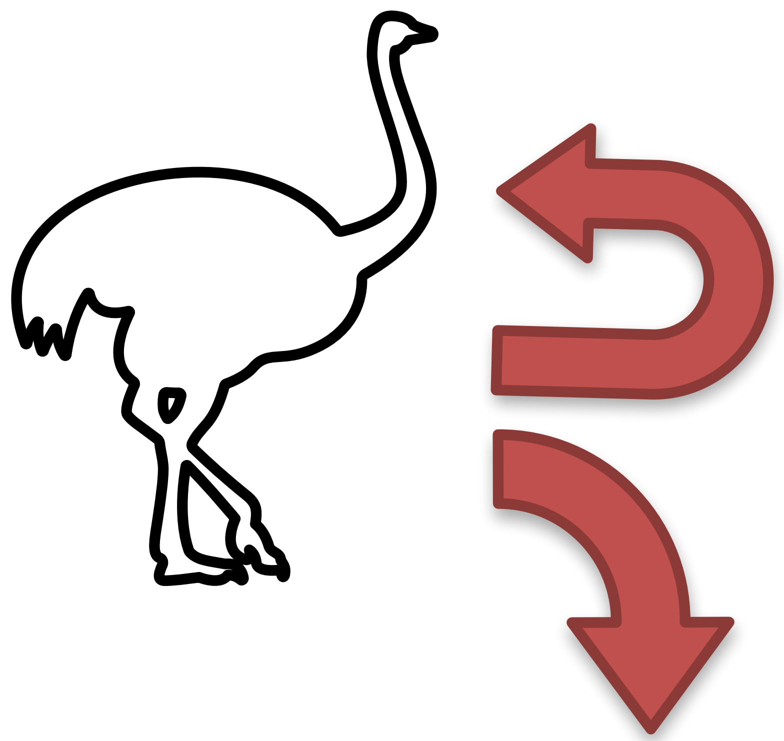

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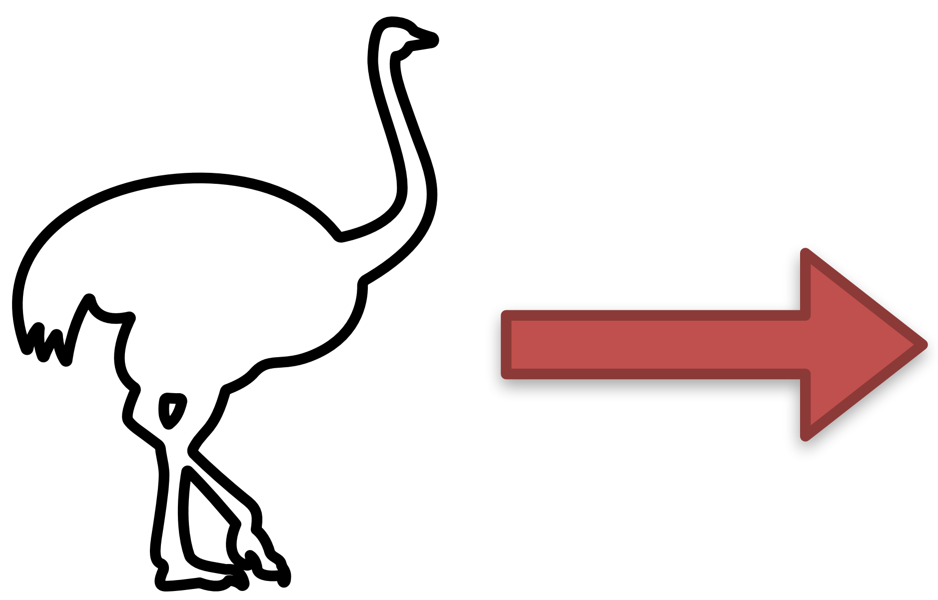

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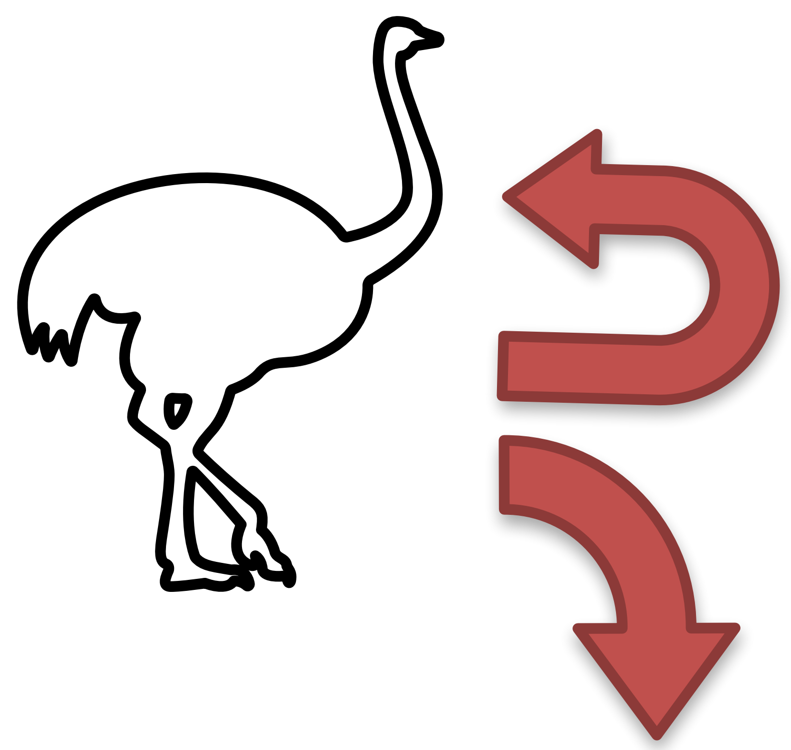

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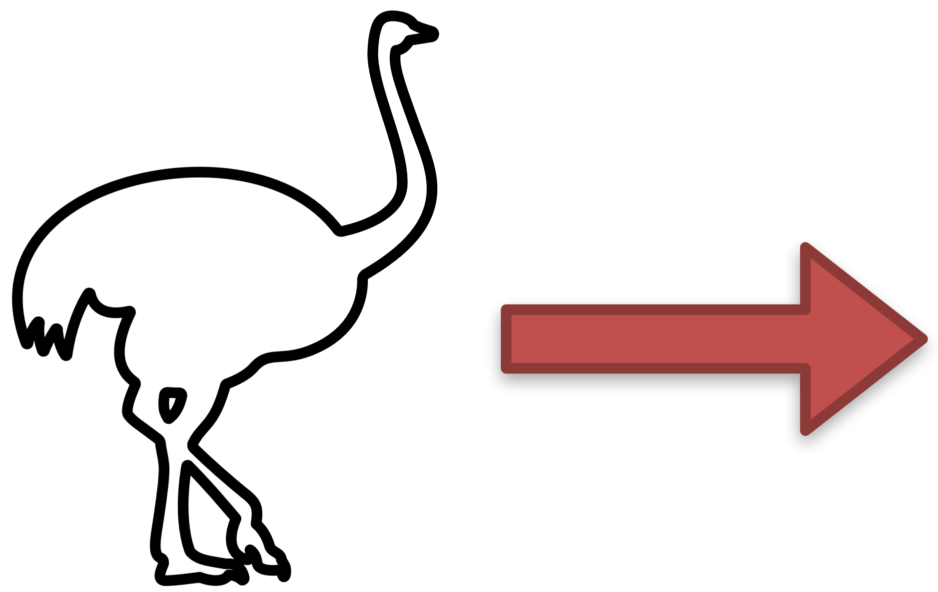
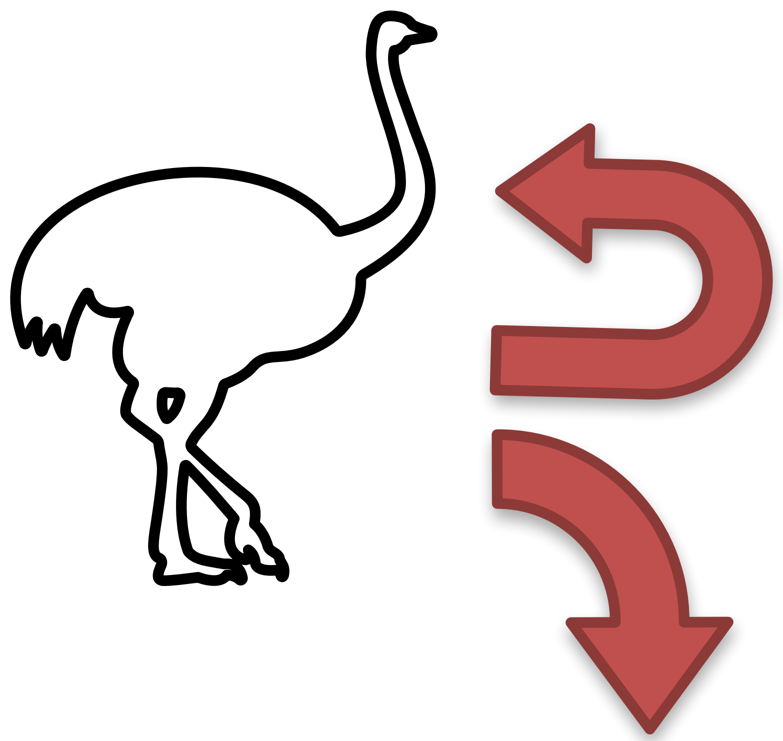


$$A_t = \operatorname*{argmax}_a Q(S_t, a)$$

$$\epsilon$$



$$A_t = \text{Random action}$$

- optimistic initial values
- R-max, MBIE
(require models)

Off-policy learning

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- Also useful for batch-RL, learning from demonstration, and parallel learning (e.g., many value functions, many policies, option-models)

Key characteristics of RL

- ☑ Evaluative feedback (reward)
- ☑ Delayed consequences
- ☑ Must associate different actions with different situations
- ☑ Online and Incremental learning
- ☑ Need for trial and error, to explore as well as exploit
- Non-stationarity

Q-learning: learning never ends

Q-learning (off-policy TD control) for estimating $\pi \approx \pi_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily

Initialize S
Loop for each step
Choose A from S using policy derived from Q (e.g., ε-greedy)
Take action A, observe R, S'
$Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma \max_a Q(S', a) - Q(S, A)\big]$
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Key characteristics of RL

- ☑ Evaluative feedback (reward)
- ☑ Delayed consequences
- ☑ Must associate different actions with different situations
- ☑ Online and Incremental learning **Bootstrapping**
- ☑ Need for trial and error, to explore as well as exploit
- ☑ Non-stationarity **Never-ending learning**

MDPs, value-functions, policies

Randomization, Off-policy

Now how do we do this with approximation?

The need for approximation

- In real world problems, **tables** of values would become intractably large
  - sometimes the state-space is too large (e.g., Go)
  - sometimes the state-space is continuous
- Instead using tables for our value functions, we will use parameterized functions
- Frame learning these approximate value functions as a supervised learning problem:
  - new challenge balancing **Generalisation** and **Discrimination**

Function approximation

- Represent the action-value function by a parameterized function with parameters $\mathbf{w} \in \mathbb{R}^n$

$$\hat{q}(s, a, \mathbf{w}) \approx q_{\star}(s, a)$$

- The approximator could be a NN, with the weights being the parameters of the network
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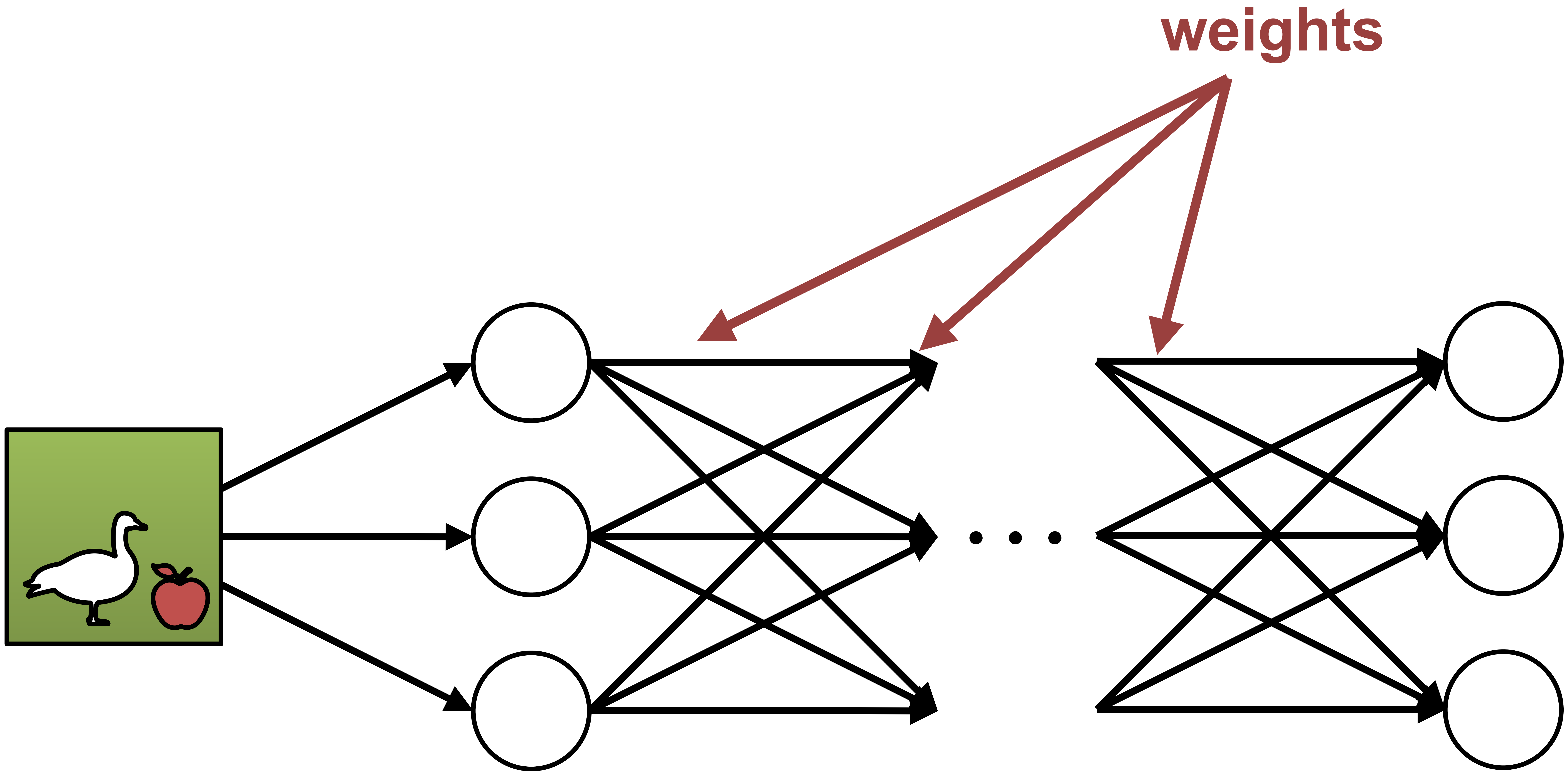
Approximating q_π with an NN



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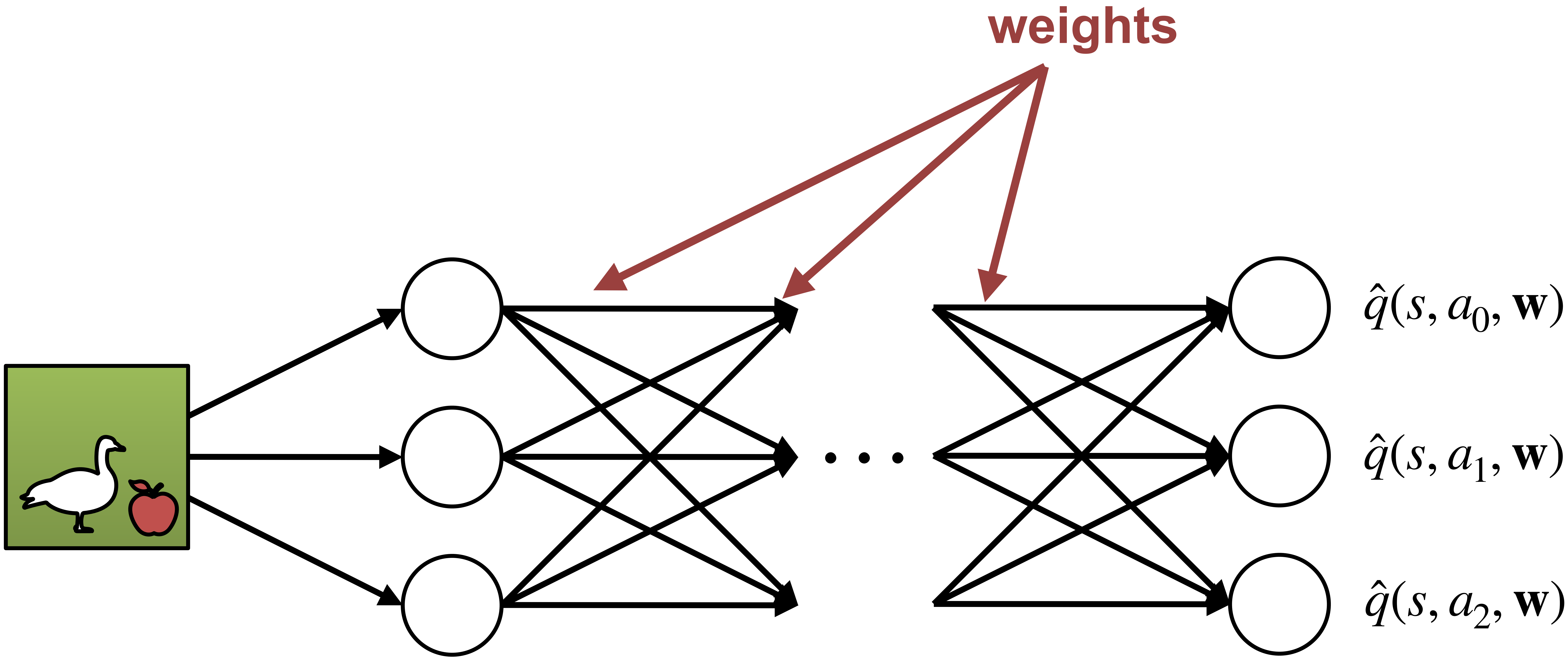

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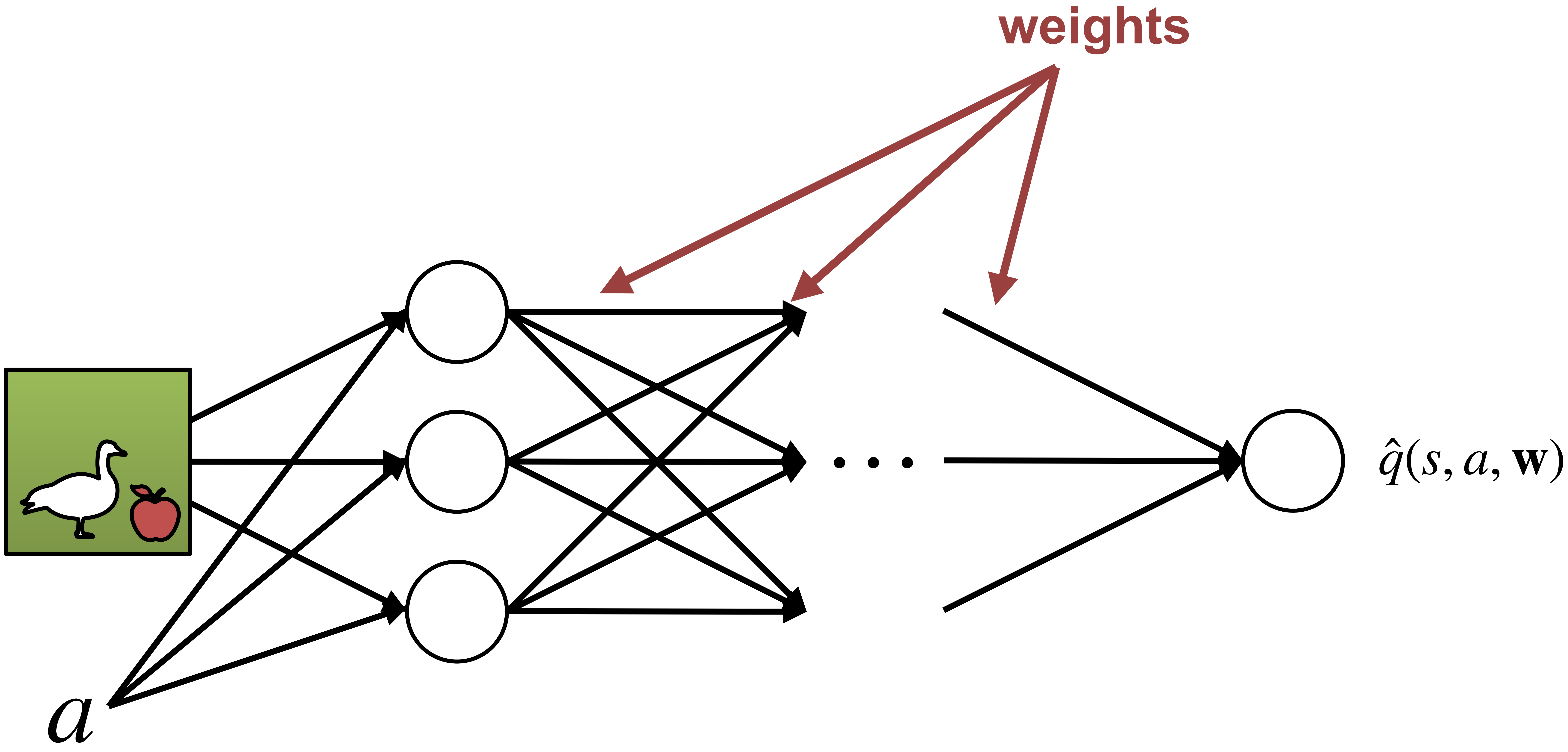

Generalization: Updates to One State Affect the Value of Other States

State	Action	Q
s_1	F	**4**
s_2	F	**-4**
s_3	F	**2**
s_4	F	**10**
s_5	F	**4**

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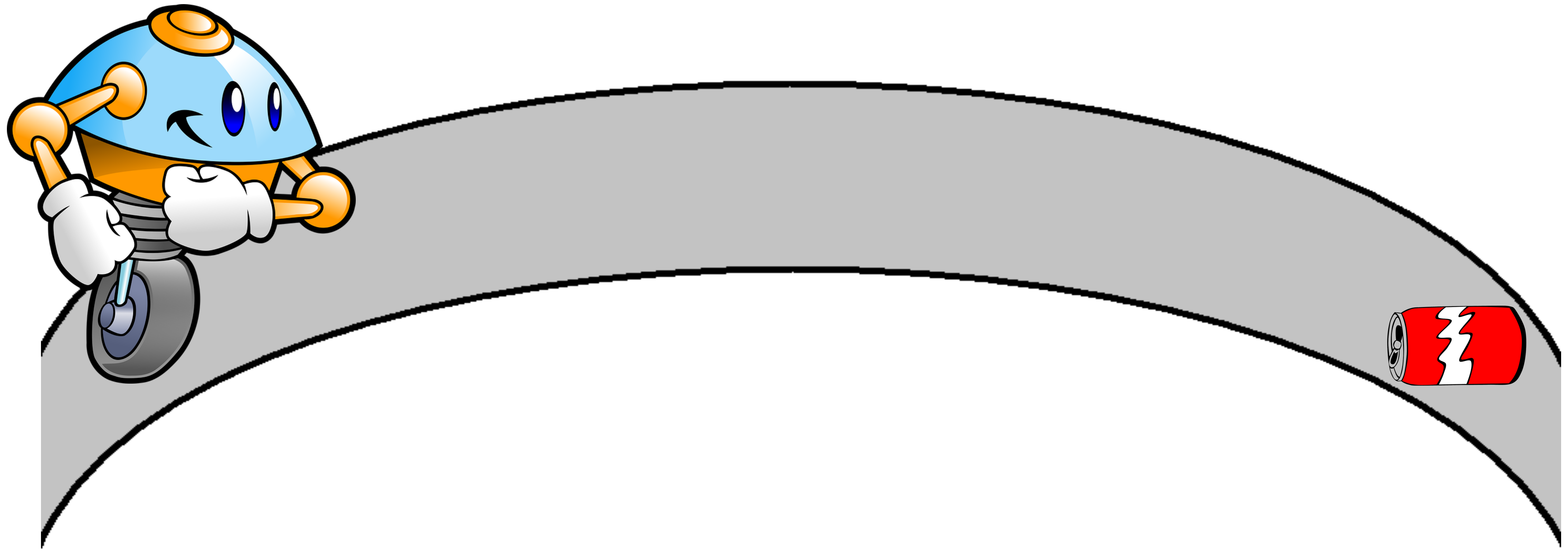

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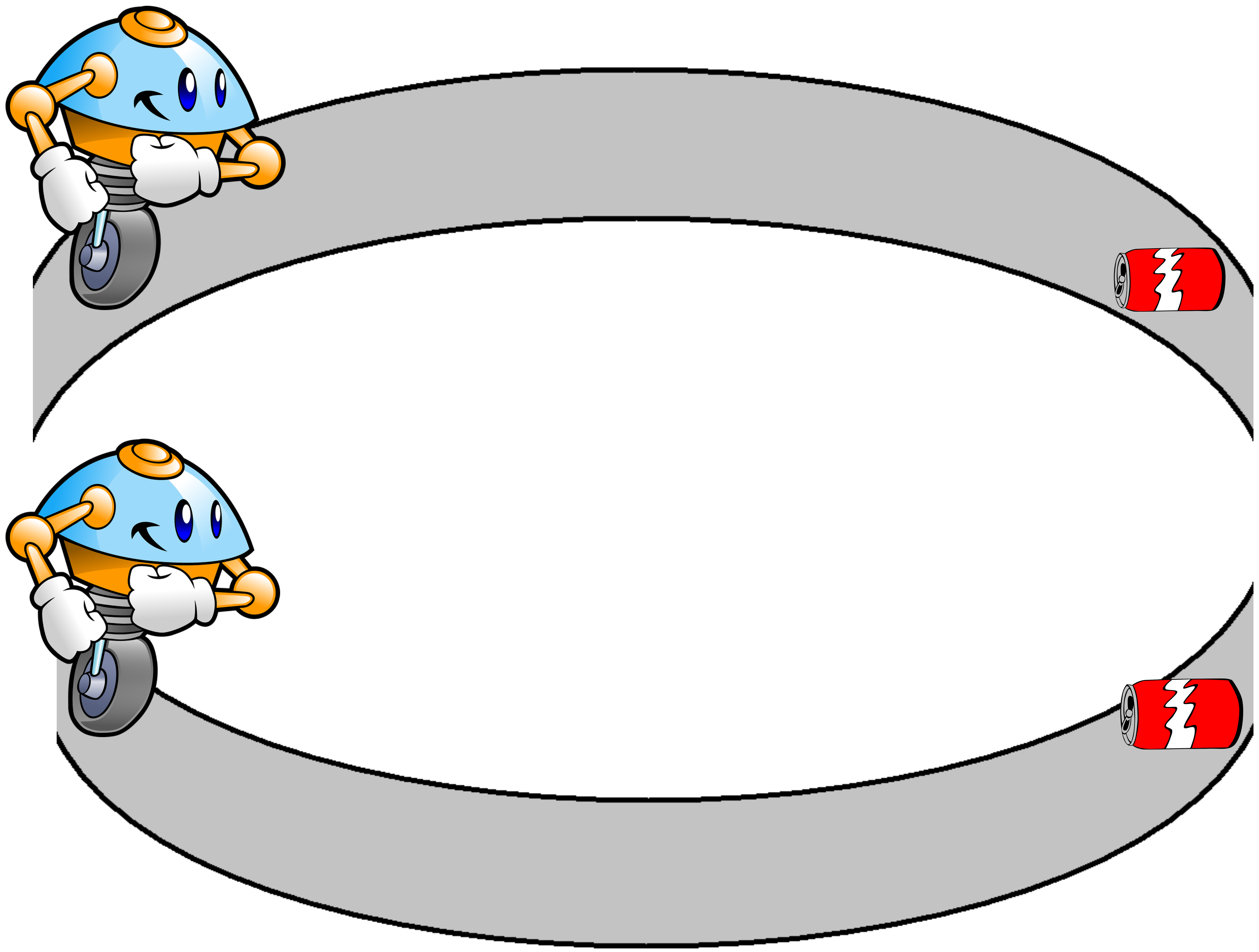

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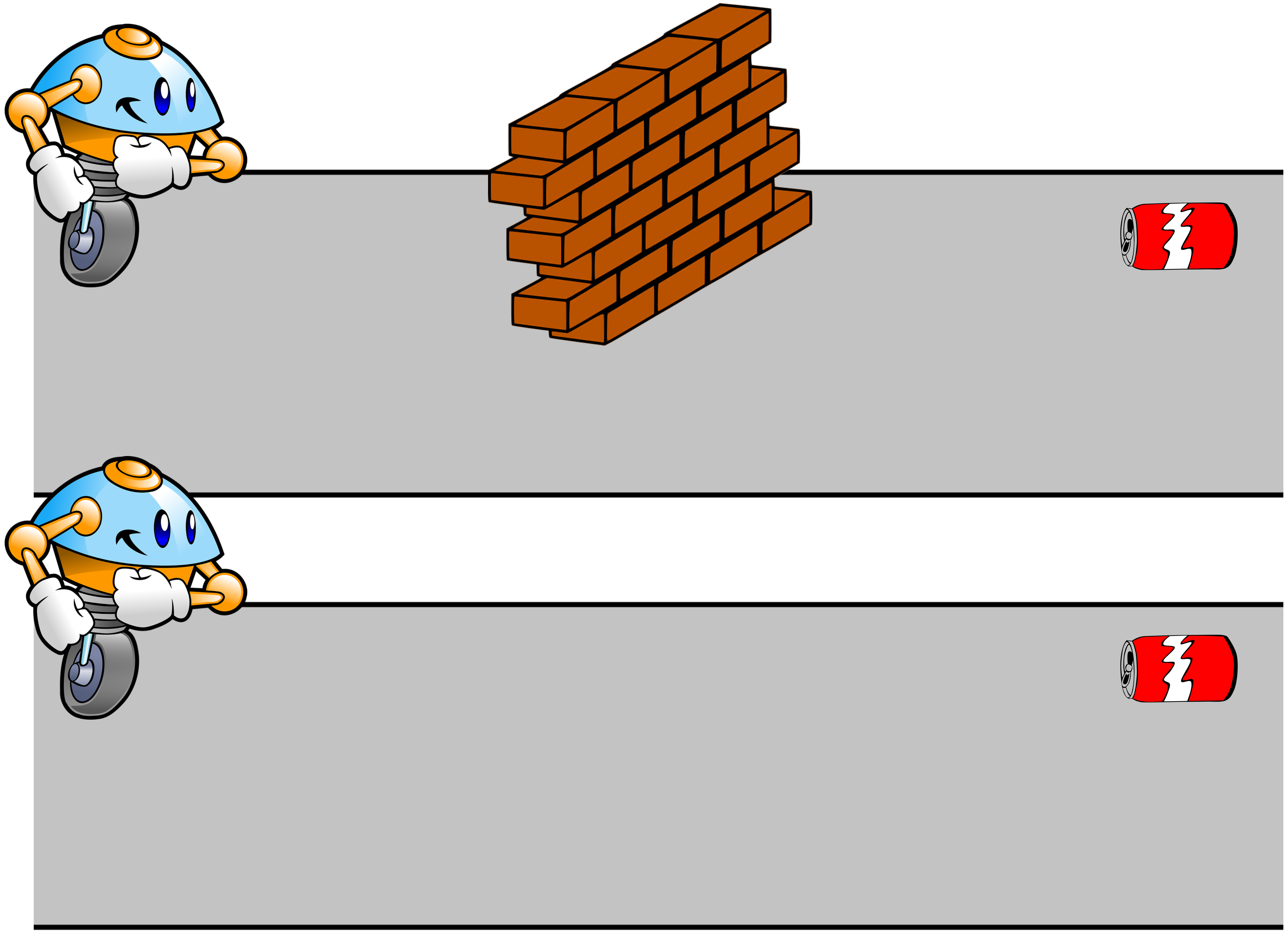
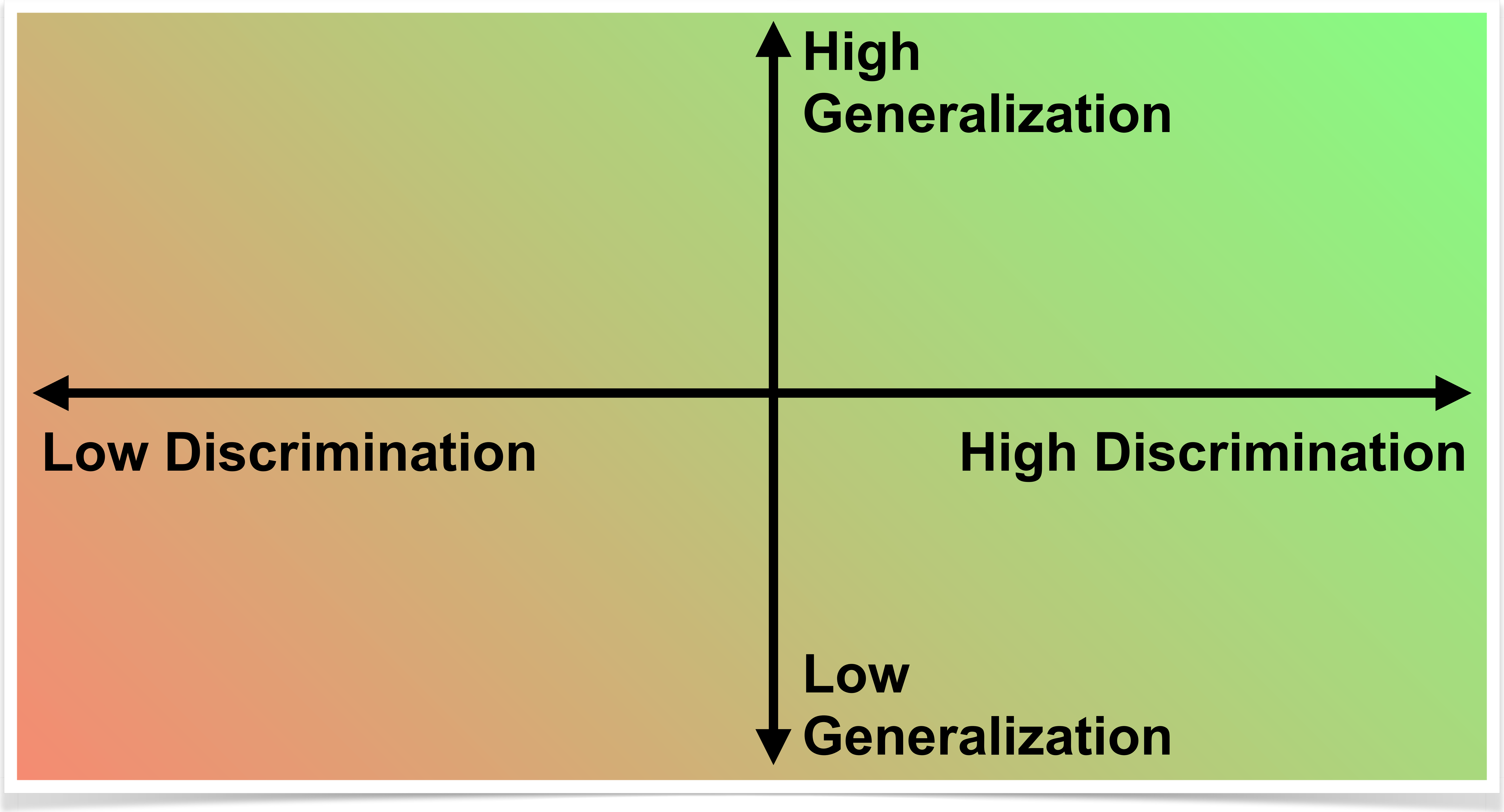


Categorizing methods based on Generalization and Discrimination



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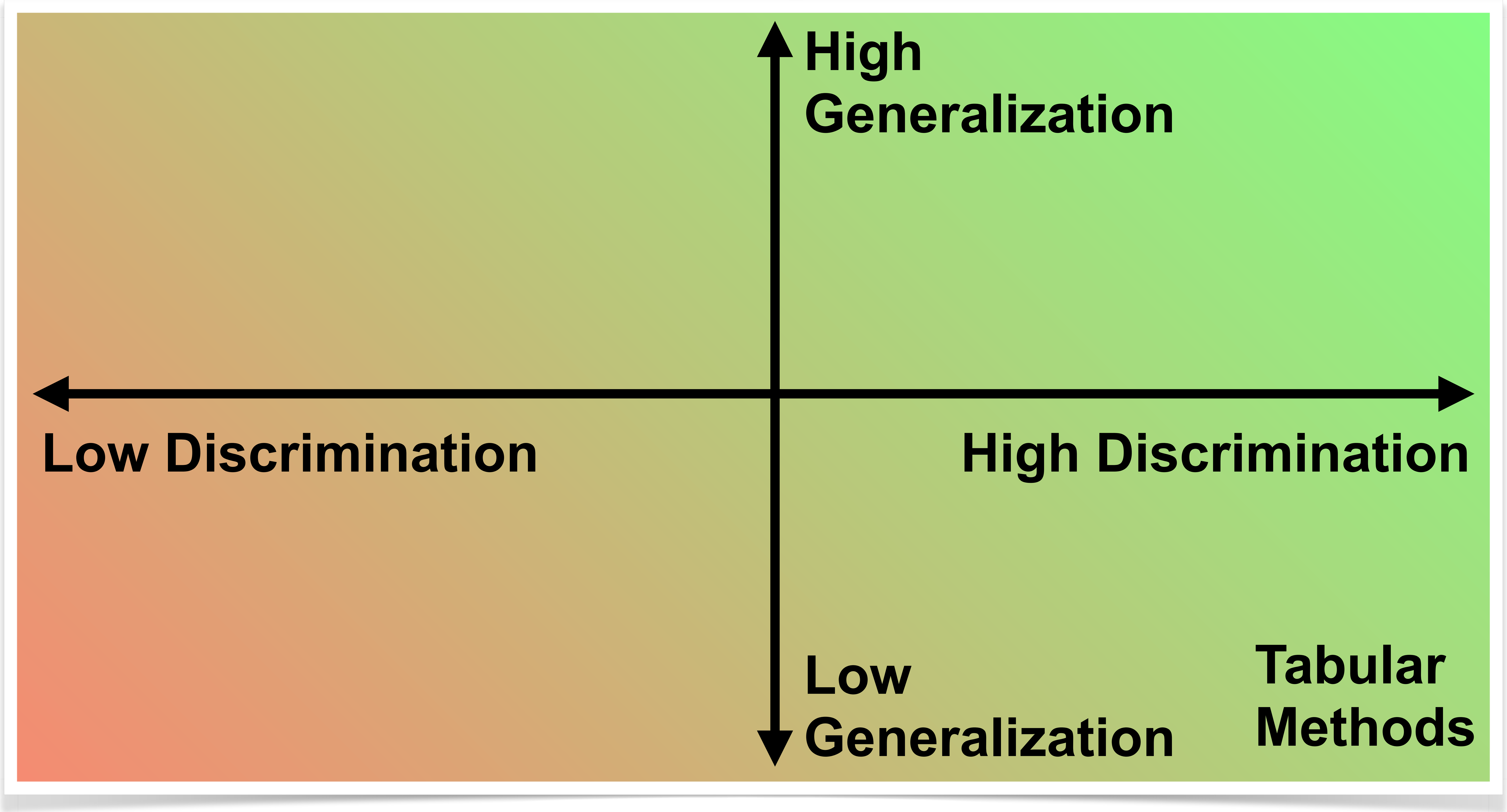

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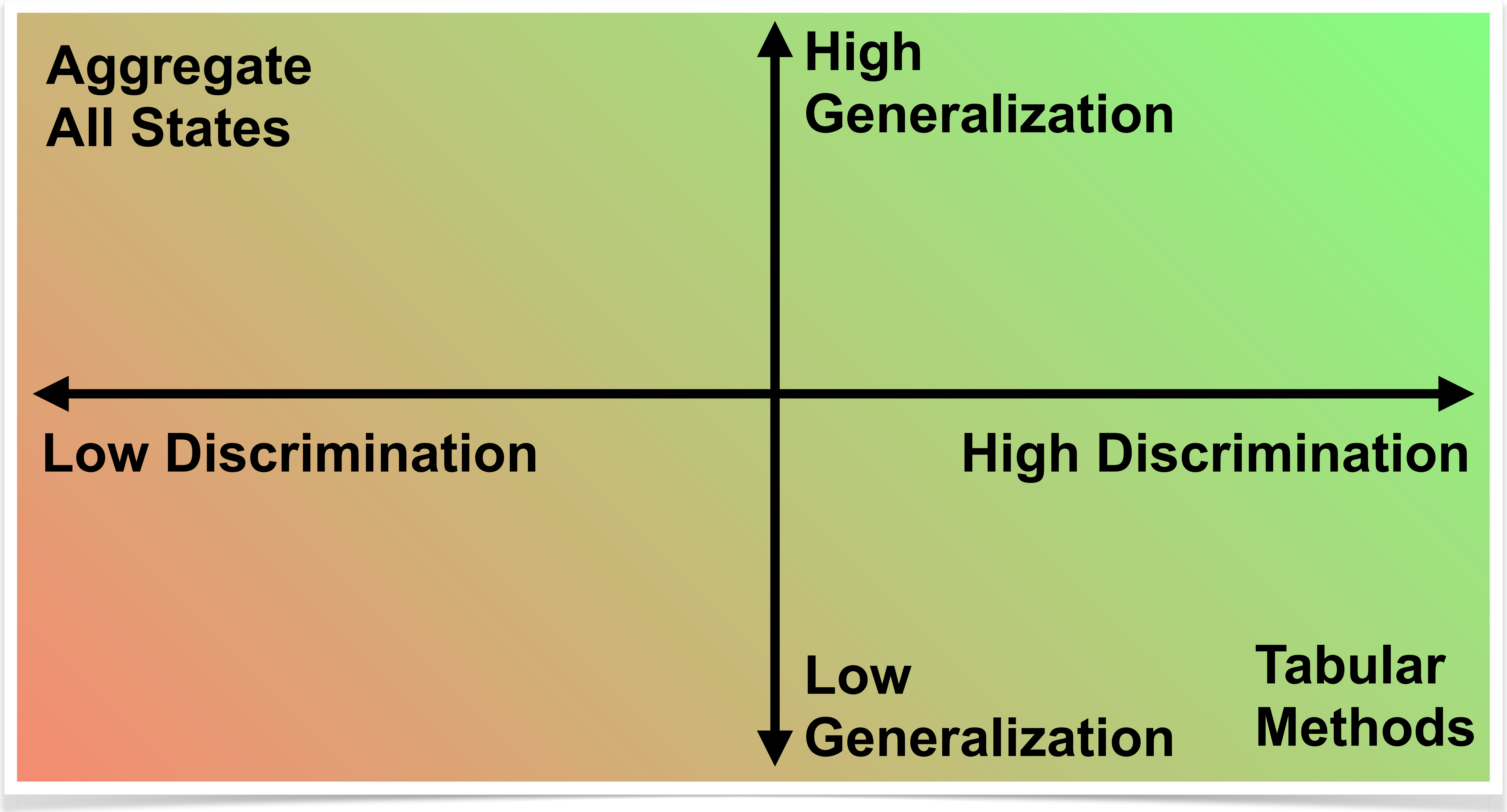

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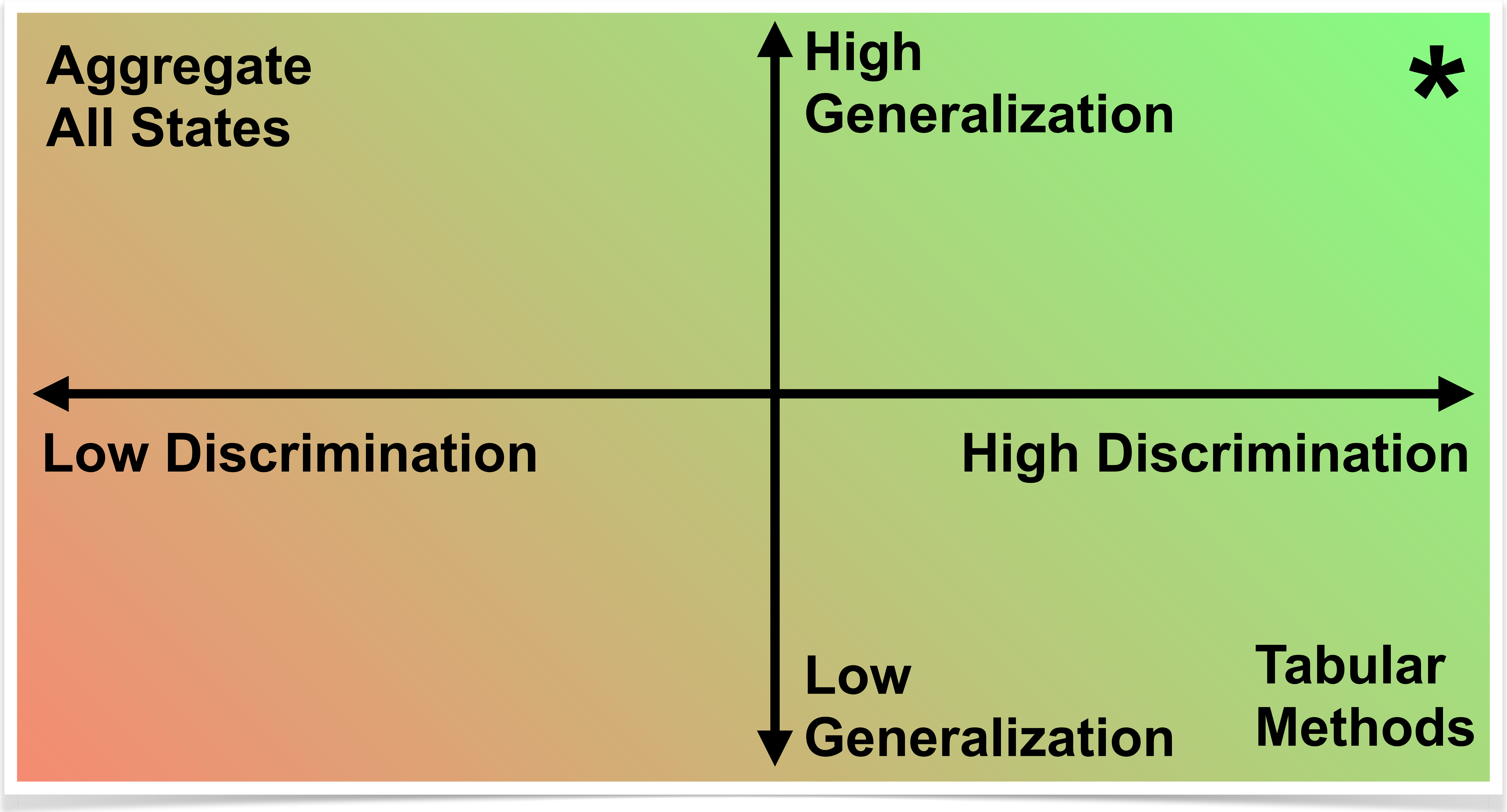

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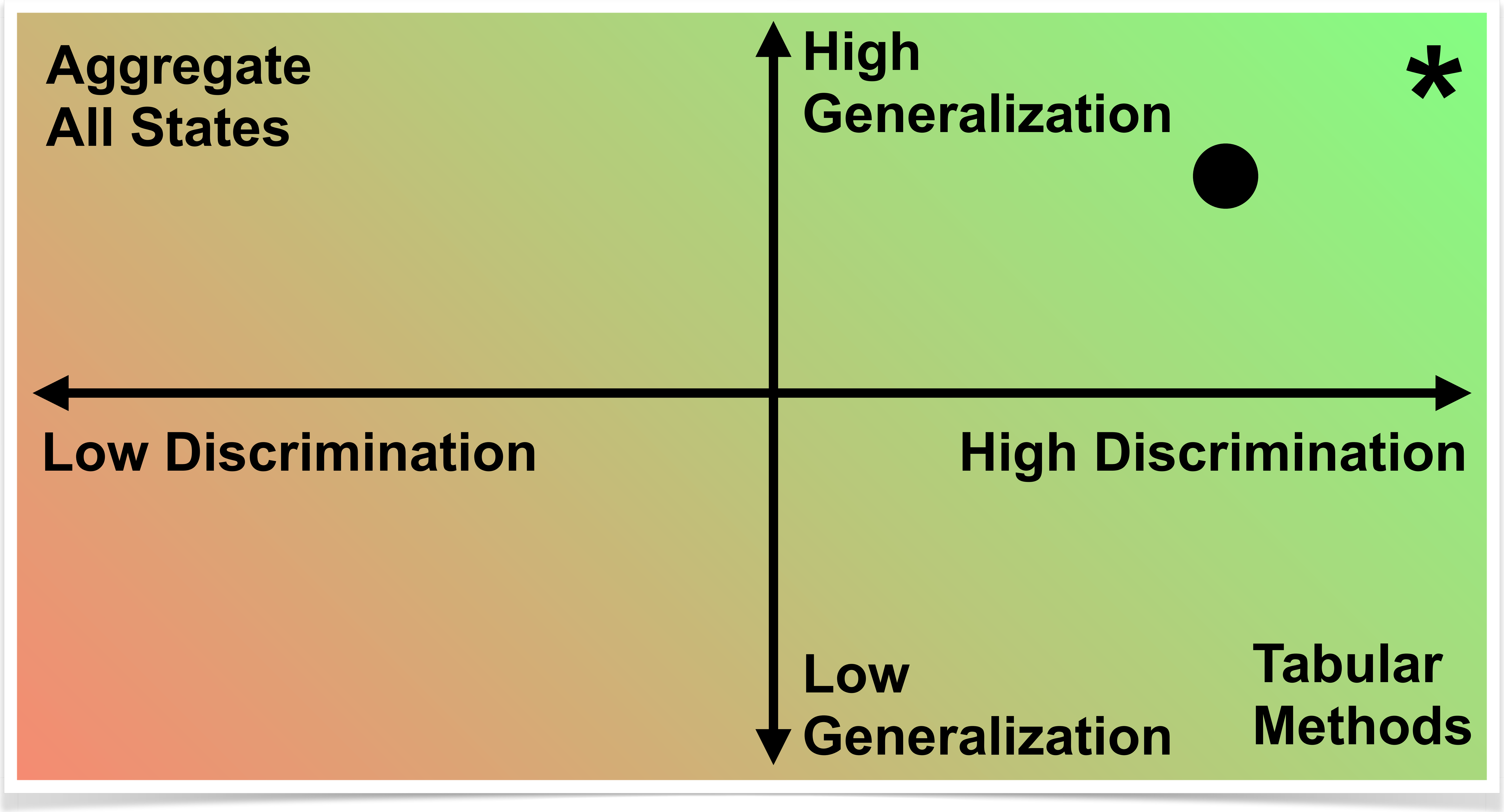

Semi-gradient Q-learning

- There is an obvious generalization of Q-learning to function approximation (Watkins 1989)

- Consider the following objective function:

$$\mathscr{L}(\mathbf{w}) = \mathbb{E}\left[\left(R_{t+1} + \gamma \max_a \hat{q}(S_{t+1}, a, \mathbf{w}) - \hat{q}(S_t, A_t, \mathbf{w})\right)^2\right]$$

- and the update used in Q-learning with function approximation

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- The target here depends on the **w.** It's like we ignored the gradient of the value of the next state

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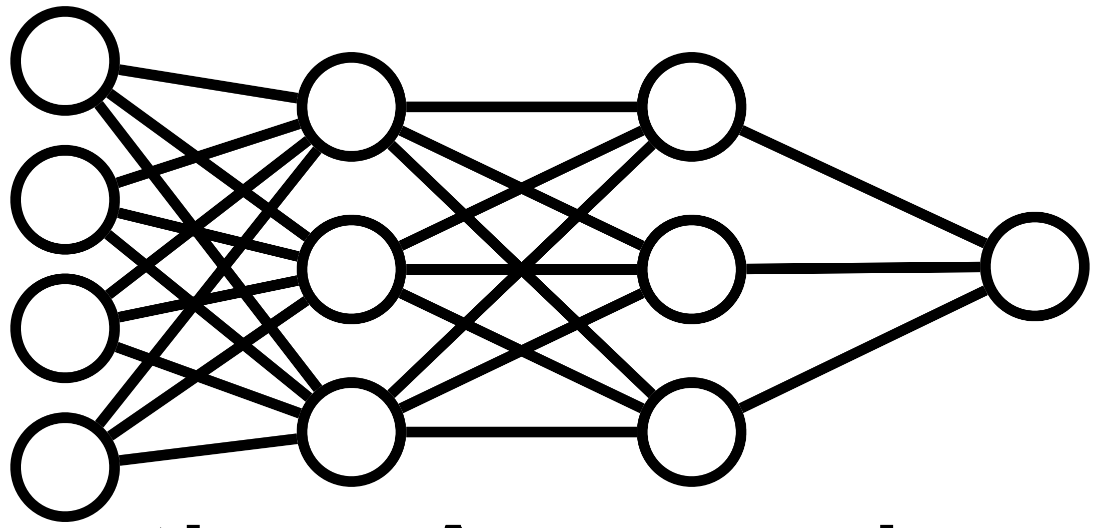
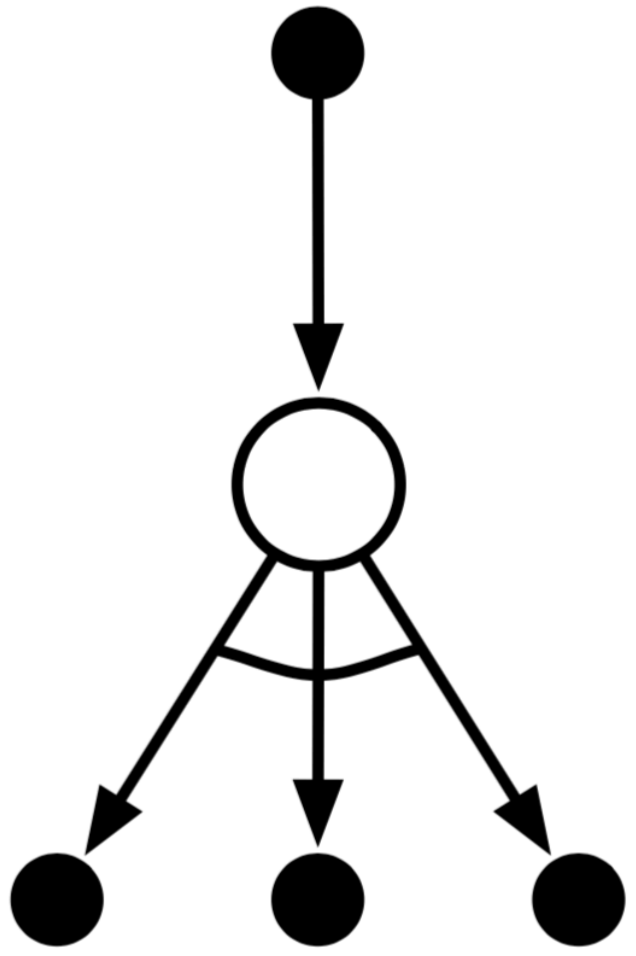
- Even TD with linear function approximation can diverge!
  (in off-policy prediction)

The deadly Triad



Function Approximation

Bootstrapping



Off-policy Learning

Algorithmic solutions to the Triad

- Newish Gradient-TD methods (TDC, GQ, proximal-gradientTD) developed by Maei (2011) and Mahadevan et al (2015) are **sound with off-policy + function approximation**
  - limited practical experience
  - basically unexplored with non-linear function approximation
- New methods to reduce variance in off-policy training (Re-Trace, V-trace, ABQ)
  - can diverge
- Divergence with control and NN is a complex story (van Hasselt et al, 2018)
  - its more likely with larger differences between the policies (common in prioritized replay, sample-based planning, parallel learning)
  - its more likely with larger networks ... both things we might want in our learning systems!

Significant progress in the application of RL

- Learned the world's best player of Backgammon (Tesauro 1995)
- Learned acrobatic helicopter autopilots (Ng, Abbeel, Coates et al 2006+)
- Widely used in the placement and selection of advertisements and pages on the web (e.g., A-B tests)
- Used by Watson to make strategic decisions in Jeopardy!, beating the best human players (IBM 2011)
- Achieved human-level performance on Atari games from pixel-level visual input, in conjunction with deep learning (Deepmind 2015)
- AlphaGo to defeat the world's best Go players (DeepMind, 2016, 2017), AlphaZero to decisively defeat all in Go, chess, and shogi

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- Lose focus on **data efficiency, parameter sensitivity, exploration challenges**
- Many of the shortcuts we take in simulations are not possible on robots

We are not done!

, while DQN was trained on only 10 M frames.

Game	ES	DQN w/ ϵ-greedy	DQN w/ param noise
Alien	994.0	1535.0	**2070.0**
Amidar	112.0	281.0	**403.5**
BankHeist	225.0	510.0	**805.0**
BeamRider	744.0	**8184.0**	7884.0
Breakout	9.5	**406.0**	390.5
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(Plappert et al, 2017)

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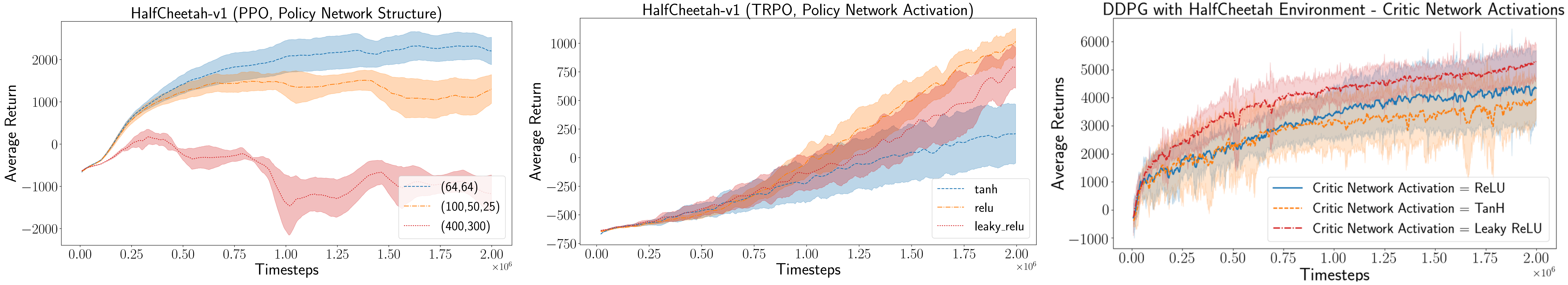

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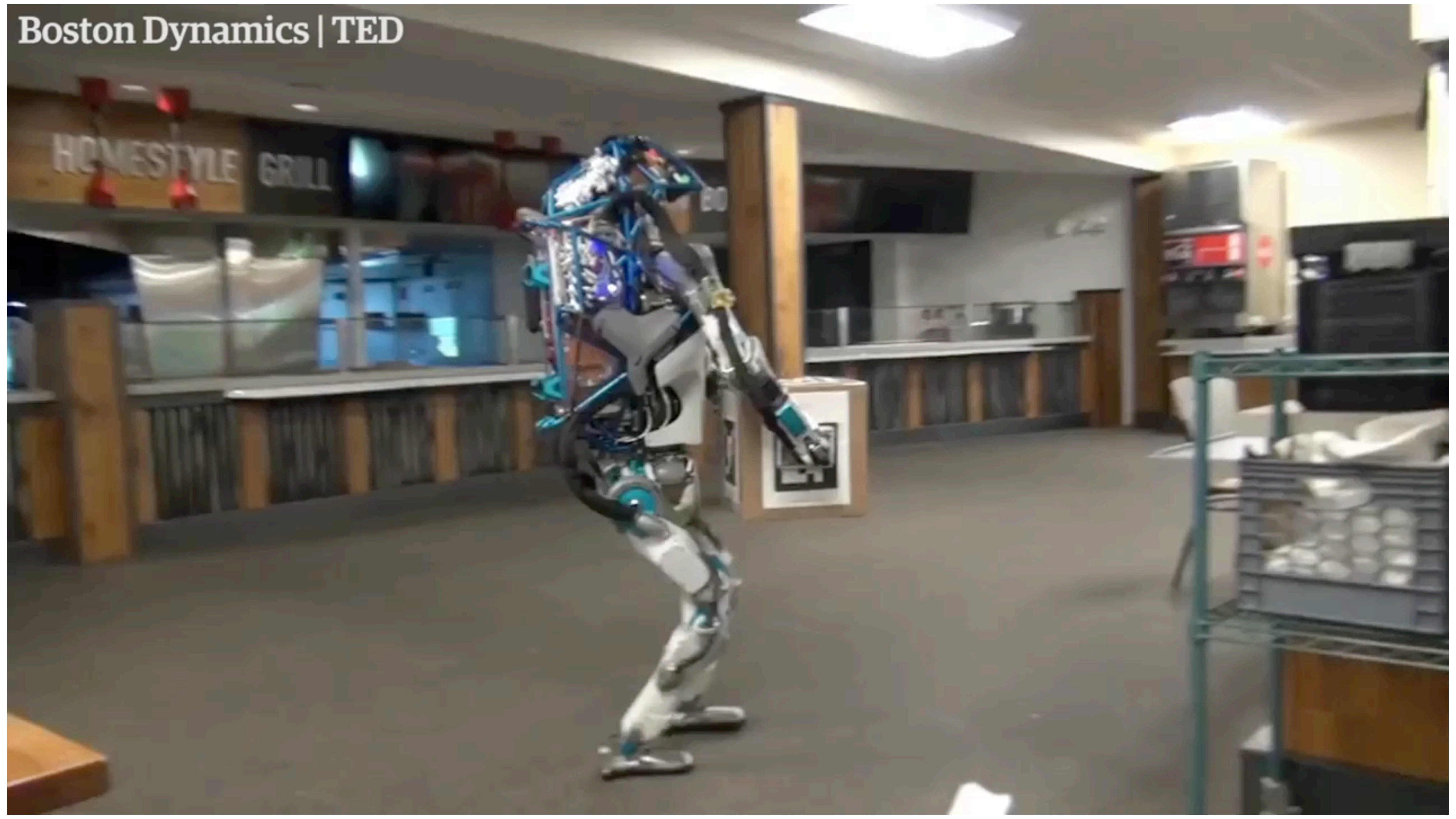

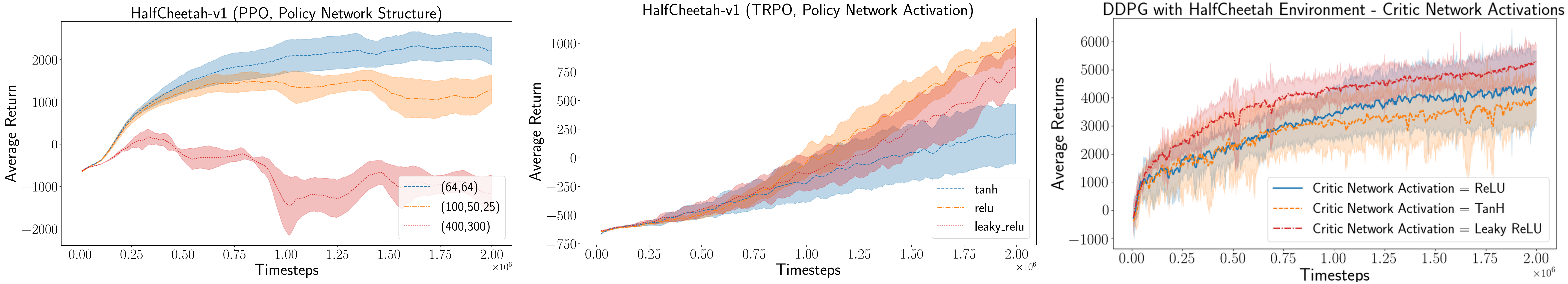

(Henderson et al, 2018)

The dimensions of RL

- Problems
  - Prediction and control
  - MDPs, Contextual-DP, Contextual Bandits, and simple Bandits
- Solutions
  - Bootstrapping and Monte Carlo (unified by eligibility traces)
  - Tabular and function approximation
  - On-policy and off-policy
  - Model-based and model-free
  - Value-based and policy-based
  - Primitive actions and temporal abstraction

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